**Project Report on**

**FRACTAL ANALYSIS OF FINANCIAL MARKETS USING MACHINE LEARNING**

***Submitted in partial fulfillment of the requirement of***

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**Bachelor of Technology**

**in**

**Electronics and Communication Engineering**

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2022

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CERTIFICATE

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in partial fulfillment for the award of **Bachelor of Technology** Degree in **Electronics and Communication Engineering, Jawaharlal Nehru Technological University, Hyderabad,** is a record of the bonafied work carried out by them during the academic year 2021-2022 under our guidance and supervision.

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This is to certify that the work reported in the present project titled **“FRACTAL ANALYSIS OF FINANCIAL MARKETS USING MACHINE LEARNING”** is a record work done by me/us in the **Department of Electronics and Communication Engineering, Sreenidhi Institute of Science and Technology, Yamnampet, Ghatkesar.**

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**ABSTRACT**

Financial investment theory requires a thorough understanding of financial market behaviour. The approaches for researching financial markets established in the 1960s and 1970s are only useful when market conditions are steady. They are based on the assumption that financial markets behave according to the normal distribution law. In the 1990s, they began looking at this topic through the lens of fractal analysis. The property of self-similarity has been discovered in financial time series. Mandelbrot (1983, 2006), the pioneer of fractal geometry, considered the behaviour of financial indicators in the market to be fractals.. E. Peters' books "Fractal study of financial markets" and "Chaos and order in the capital markets" describe this topic's research. The current situation of financial time series in the stock market is the subject of the research given here. Financial time series are examined as fractals in this study. The volatility and persistence of series are explored. By mixing the series, the persistence hypothesis was tested once again for the persistent series. The average lengths of non-periodic cycles in these series were also discovered.

Keywords: Fractal analysis; Fractal time series; Financial market; Volatility of financial series.

###### CONTENTS

**Abstract i**

[Contents ii](#_TOC_250063)

List of figures a-b

List of Tables c

[CHAPTER 1 INTRODUCTION 1-16](#_TOC_250062)

* 1. [An Overview of Fractal geometry and Econophysics 2](#_TOC_250061)
  2. [Stylized facts 6](#_TOC_250060)
  3. Standard Econometric models
  4. Introduction to Multifractality
  5. [Motivation 16](#_TOC_250043)
  6. [Objective 16](#_TOC_250042)

[CHAPTER 2 LITERATURE SURVEY 16-36](#_TOC_250041)

CHAPTER 3 MULTIFRACTAL ANALYSIS 37-56

* 1. [Partition function approach 37](#_TOC_250040)
  2. [Structure function approach 37](#_TOC_250039)
  3. [Wavelet transform approach 38](#_TOC_250038)
  4. Detrended fluctuation approach 39
  5. Other methods 39
     1. [Multiplier method 40](#_TOC_250023)
     2. [Multifractal Hilbert-Huangspectral analysis 42](#_TOC_250023)
     3. [EMD-based multifractal methods 43](#_TOC_250023)
     4. [Multiscale multifractal analysis 44](#_TOC_250023)
     5. [Time-varying multifractal analysis 45](#_TOC_250023)
     6. [Phase space reconstruction 46](#_TOC_250023)
     7. [Asymmetric multifractal analysis 47](#_TOC_250023)
     8. [Multifractal diffusion entropyanalysis 48](#_TOC_250023)
     9. [Symbolization presentation and Renyi dimensions](#_TOC_250023)  49

CHAPTER 4 PROPOSED TECHNOLOGY 50-79

* 1. Emperical Evidence of multifractal markets 50
  2. Volatilities 51
     1. [Stockmarkets 52](#_TOC_250016)
     2. [Foreignexchangerates 53](#_TOC_250016)
     3. [Commodities](#_TOC_250016) 55

4.3 Data and Methods 57

4.4 Accuracy 58

**CHAPTER 5 RESULTS AND DISCUSSIONS 59-65**

**CHAPTER 6 CONCLUSIONS 66-69**

**References 70-76**

###### Appendix

**LIST OF FIGURES :**

Figure 1: Time series of two stock market indices' daily

closing prices (left), daily returns (centre), and daily

volatilities (right). The Dow Jones Industrial Average index is

in the upper panel, while the Shanghai Stock Exchange

Composite index is in the lower panel. 17

Fig1.2 Time series of logarithmic return of CSI300 18

Fig.1.3 Time series of logarithmic return of S&P 500 index 18

Fig.3.1 Multifractal analysis 20

Fig.3.2 Multifractal analysis of daily prices 20

Fig.3.3 Extended multifractal analysis 21

Fig.4.1 Algorithm 58

Fig.5.1 Result comparision of empirical and theoretical values 65

**LIST OF TABLES**

Table.3.1 Result based on autocorrelated values 68

#### CHAPTER-1

#### INTRODUCTION

* 1. **An Overview of Fractal geometry and Econophysics**

Physicists have forever been fascinated by money markets, each professionally and academically. He famously quipped of his theoretical studies, "I can calculate the motions of celestial bodies, but not the craziness of people." The Prediction Company was created in 1991 by Doyne Farmer, Norman Packard, and James McGill to explore computerised statistical arbitrage strategies. Prediction Company was such a hit that it was sold to UBS in 2006 and subsequently to Millenium Management in 2013. Science & Finance, a research organisation founded by Jean-Philippe Bouchaud and Didier Sornette in 1994, merged with Capital Fund Management in 2000. There have been a number of other instances of scientists flirting with Wall Street, with Emmanuel Derman being one of them [1].

Eugene F. Fama, an economist, proposed the efficient market hypothesis (EMH) in 1970, which has since become a cornerstone of modern finance theory. According to this idea, all market information is immediately reflected in the stock price, making stock prices unpredictable [1]. Later behavioural finance theory investigations of market behaviour have shown the EMH's weaknesses. Investor irrationality, market friction, and imperfect arbitrage are cited as examples of violations of the effective market hypothesis [2–4]. Financial time series have been observed to have momentum effects, reversal effects, January effects, and financial anomalies such as peaks and thick tails of time series, volatility clustering, and so on [5–9]. As a result, economists have been actively looking for new hypotheses to explain these market inconsistencies. Peters presented the fractal market idea in 1994. (FMH). The efficient market hypothesis is modified by this hypothesis, which states that asset prices follow fractional Brownian motion, the return rate sequence has long memory, and the market may be out of equilibrium [10]. As a result, a certain amount of pricing predictability has gained widespread acceptance. After the FMH was proposed, the field primarily focused on two aspects: studying the stock market's fractal properties and developing various models to anticipate market changes.

The rescaled range approach (R/S) devised by British hydrologist Hurst [11] is the starting point for the first aspect, which is the way of examining fractal characteristics. He discovered that a biassed random walk (fractional Brownian motion) can well describe the long-term dependence of the two when studying the relationship between the Nile Reservoir discharge and the water level, so he proposed calculating the Hurst exponent using the rescaled range method, which was used to characterise the self-similarity of time series. Many academics have worked to refine and perfect the approach throughout time.

Physicists and economists have been motivated by the fundamental philosophical linkages (as well as major discrepancies) between physics and finance [2, 3]. Financial markets are viewed by physicists as complex systems in general, and as a result, several scientific research have been conducted. The 1990s planted many of the seeds of today's blooming econophysics. In the last two decades, four key study lines in the field of econophysics have emerged as the most active. In 1991, Rosario Mantegna and Gene Stanley investigated the Levy-walk-like super-diffusive -behaviour of Milan stock market indexes [15] and the scaling behaviour of the S&P 500 index in 1995 [16, 17]. In 1996, Ghashghaia et al. looked at the multifractal behaviour of the structured function of the US dollar vs German mark exchange rates [18].

Kantelherdt et al. pointed out that in most situations, time series scaling behaviour is complex and cannot be characterised by a single scaling index [13]. As a result, the multifractal detrending volatility approach (MF-DFA) was presented, with the author pointing out that the thick-tailed distribution and long-range correlation might produce a multifractal structure. Thompson et al. analysed the fractal properties of GE stock price data using many approaches. The MF-DFA model was shown to be more well-fitting [14]. Multifractals are frequent in financial markets in numerous nations, according to a number of studies, including stock markets [15–19], bonds [20], and Bitcoin markets [21]. The facts presented above categorically refute the efficient market theory. Some academics, however, have questioned the MF-DFA approach. Bashan pointed out that the MF-DFA may result in spurious fluctuations, which may be represented in the bigger estimated generalised Hurst index [22] after comparing DFA, CMA, MF-DFA, and other detrend volatility analysis methods. The fitting polynomials of neighbouring intervals may be discontinuous because the intervals separated using the MF-DFA approach do not overlap. Recognizing this flaw, several researchers improve the model using overlapping smoothing windows, which lowers the false oscillations induced by partially overlapping neighbouring periods [23,24]. This optimization approach, known as OSW-MF-DFA, is used. Some researchers looked at the multifractal changes in the financial sector as a result of the pandemic and found that COVID-19 reduced market efficiency [25–27]. Okorie investigated the stock market fractal's contagion effect, which demonstrated the presence of multifractals from a different perspective [28].

Although stock markets are influenced by a variety of factors such as macroeconomic development, institutions, supervision, and noise trading, researchers continue to try to build various prediction models: from parametric models such as ARMA, ARIMA, and GARCH to machine learning models such as BP, recurrent neural network (RNN), LSTM, and GRU with gated structure, the prediction accuracy of the model has been continually improved. Hochreiter and Schmidhuber proposed the short-term memory neural network (LSTM) in 1997 [29]. The gated recurrent networks LSTM and GRU, which have been famous in recent years, have been frequently utilised to forecast stock price trends, and they have shown to be effective at catching the long and short term memories of financial time series [30–32]. The GARCH model and the LSTM neural network were employed by Yu et al. to forecast the volatility of China's three major stock indexes, and the findings showed that the LSTM with long memory had greater predictive performance [33]. However, as LSTM models have become more popular, additional research have discovered drawbacks such as limited explanatory power and sluggish convergence time. To address the drawbacks of LSTM, Cho et al. suggested a GRU neural network [34], which was further tuned on the basis of LSTM. Unlike LSTM, GRU only has two gate control structures: update gate and reset gate, which minimises parameters while retaining predictive performance and aids convergence [35,36].

Stock markets are attracting an increasing number of participants, including financial experts, casual traders, and analysts for global corporations and government agencies. Events in the stock market can be analysed in a variety of ways. One of the methods for analysing financial time series is fractal analysis. Since the turn of the century, fractal analysis has been utilised to investigate financial markets. Financial time series with the property of self-similarity were first identified as fractals. Long-term financial time series, also known as persistent financial time series, are the subject of this research. From a commercial sense, such programmes are very enticing. They are more predictable because they recall previous knowledge while evaluating the indications that follow. Two of the authors' financial series have been shown to be persistent [1]. They figured out the average durations of non-periodic cycles, which are important in financial series analysis and investing. The term "fractal geometry" was coined by Benoit Mandelbrot, who was the first to use it. Fractus, which meaning "broken" in Latin, is the root of the word fractal. 'To shatter,' as in 'to make irregular fragments,' is the Latin term frangere.

Fractus can also imply 'irregular,' in addition to 'fragmented.' The term fragment encompasses both meanings. As a result, the term "fractal" is an excellent choice for describing the study of scale invariant or self-similar geometric objects. similar characterized ones are an important property of natural things' geometry, which is both fundamental and universal. This is referred to as'self-similarity,' which means they appear as similar at different levels of scale. Otherwise, they are self-affine or have an affinity at various scales[2]. The self-affinity of natural objects is demonstrated not just by their shape or geometry, but also by their chronological history. Financial time series and the temporal evolution of a self-allined stochastic field are examples of the latter condition.

The Fractal Market Hypothesis is based on the fact that a time series of financial market seems to be similar statistically on many time periods. Price value distributions over a single day, for example, are same as those over a month, which are same as to those over a year, and so on (by assuming there is sufficient data over every time scale to make the distribution—the resulting statistically significant histogram). Fractal geometry's mathematical origins may be traced back to 17th-century books on recursion. Research in the nineteenth century on classes of continuous functions that were not differentiable in the classical sense aided this finding. Benoit Mandelbrot investigated the roots of self-similar geometry in his work 'Similarity and Fractional Dimension in Statistics.' In 1975, he coined the term "fractal," which included the concepts and outcomes of hundreds of years of mathematical research.

Stock markets are attracting an increasing number of participants, including financial experts, casual traders, and analysts for global corporations and government agencies. Events in the stock market can be analysed in a variety of ways. One of the methods for analysing financial time series is fractal analysis. Since the turn of the century, fractal analysis has been utilised to investigate financial markets. Financial time series with the property of self-similarity were first identified as fractals. Long-term financial time series, also known as persistent financial time series, are the subject of this research. From a commercial sense, such programmes are very enticing. They are more predictable because they recall previous knowledge while evaluating the indications that follow. Two of the authors' financial series have been shown to be persistent [1]. They figured out the average durations of non-periodic cycles, which are important in financial series analysis and investing. The term "fractal geometry" was coined by Benoit Mandelbrot, who was the first to use it. Fractus, which meaning "broken" in Latin, is the root of the word fractal. 'To shatter,' as in 'to make irregular fragments,' is the Latin term frangere.

We intend to utilise the GRU model to anticipate the fractal features of the Chinese and American intraday stock markets as a result of the COVID-19 outbreak. Based on the impact of the virus on the financial markets, we divided the time period into three phases: before, during, and after the pandemic's first panic period. For multifractal research, this article employs the OSW-MF-DFA method, which is enhanced by overlapping smoothing windows. The two stock indexes' generalised Hurst index and multifractal spectrum are calculated, and the fractal features of the two markets are examined and contrasted across time.

We discovered through practical studies that the two markets are always multifractal, and the degree of fractal intensifies throughout the pandemic. The pandemic had the greatest impact on the US market. We discovered that using time-varying Hurst and its decomposition sequence as inputs to the prediction model may considerably enhance the model's prediction performance and beat volatility indicators during the panic phase of pandemics (COVID-19), which has strong volatility clustering. In the GRU neural network, a time-varying Hurst sequence is added to conventional input variables like starting price, closing price, highest price, lowest price, and volatility to see whether it will increase forecasting efficiency.

Financial professionals, casual traders, and analysts for multinational firms and government agencies are among the growing number of participants in stock markets. Stock market events may be analysed in a variety of ways. Fractal analysis is one of the ways for analysing financial time series. Fractal analysis has been used to examine financial markets since the turn of the century. Fractals were initially recognised as financial time series having the attribute of self-similarity. This study focuses on long-term financial time series, often known as persistent financial time series. Such programmes are particularly appealing from a business standpoint. They are more predictable since they recollect prior information when assessing the subsequent signs. The persistence of two of the authors' financial series has been demonstrated [1]. They calculated the average durations of non-periodic cycles, which is useful in financial series analysis and investment. Benoit Mandelbrot developed the phrase "fractal geometry," and he was the first to utilise it. The word fractal comes from the Latin word fractus, which means "broken." The Latin phrase frangere means 'to shatter,' as in 'to produce uneven fragments.'

* 1. **Stylized facts**

Financial markets are complex adaptive systems in which universal and non-universal statistical rules emerge at the macroscopic level from the self-organized microbehavior of heterogeneous traders in response to external inputs [40, 41, 54–59]. These criteria might be investigated using a large dataset of financial stock market price time series. In the left panel of Fig. 1, the daily price trajectories I(t) of two stock market indices are displayed, the Dow Jones Industrial Average (DJIA) index from 26 May 1896 to 29 December 2017 and the Shanghai Stock Exchange Composite (SSEC) index from 19 December 1990 to 29 December 2017. These two stock markets are indicative of both developed and emerging markets, both of which are hotly debated. Over a time period t, the logarithmic return of I(t) is defined by

r∆t(t) = ln I(t) − ln I(t − ∆t). (1)

In the middle panel of Fig. 1, the daily return time series r(t) with t = 1 day is presented, while the corresponding volatility time series is shown in the right panel. The volatility for a given day is defined as the square root of the sum of the squares of the intraday returns for that day, where the intraday time scale must be small enough (say 1 minute) to guarantee that the volatility estimator converges successfully. Volatility clustering is evident in the return time series, which exhibits strong bursty patterns. Several statistical regularities or stylized facts have been found in several financial markets [60–63]. We'll start by reiterating three important stylized realities that are common to all financial markets. The first stylized truth is the fat-tailed distributions of financial returns. In asset pricing and risk management, the form of the asset price fluctuation distribution is critical [25, 54, 55]. According to early empirical and theoretical research [64, 65], the essential assumption of the Black-Scholes option pricing model [66] is that asset values follow geometric Brownian motions. Incomes and speculative price returns, according to Mandelbrot in the 1960s, follow a Pareto-Levy distribution with power-law tails.

Pr(|r|) ∼ |r|−γ−1, (2)

whose exponents 1 2 stated to be in the region of stable Levy laws' attraction [6, 7, 9–11]. Remember that Levy laws generalise the Gaussian distribution, and their family includes all stable under convolution distributions with no or only mild dependence between random variables.

Diagram

Description automatically generated

Figure 1: Time series of daily closing prices (left), daily returns (middle) and daily volatilities (right) of two stock market indices. The upper panel is for the Dow Jones Industrial Average index and the lower panel is for the Shanghai Stock Exchange Composite index.

Financial economists like Fama [67], Fama and Roll [68], Samuelson [69], Sargent [70], and others were immediately intrigued by the possibility of financial gains being distributed using Levy rules rather than Gaussian distribution. People like Cootner [71] and Granger and Orr [72] raised significant concerns to discarding the statistical theory that existed for the normal case but was lacking for the other members of the Le vy laws. Further research found that, while return distributions are fat-tailed, they are not so heavy as to be described by Le vy laws: increasing and unavoidable evidence that the variance of the return is not infinite (and thus the exponent is greater than 2), thereby irreversibly excluding the heavy tail regime of Le vy laws with tail exponent less than 2 [73–76]. After all, variance and covariance-based approaches might be used. This created a void in interest in fat-tailed distributions, which was filled in the early 1990s by econophysicists' re-discovery [15–17].

Greater recent empirical investigations with more data [20– 22] appear to settle around 3 (or, more conservatively, 2 4). More empirical work on financial returns at various time scales has been done, with results ranging from exponential to stretched exponential to power law distributions [19, 77–94]. The fat-tailed nature of return distributions at small sizes with > 2 implies that the Gaussian distribution is the zone of attraction at big scales, according to these data. As a result, a progressive change from a power law-like regime at small time scales to a Gaussian regime at large time scales might be expected [18]. The stretched exponential distribution can be used to link the exponential and power-law distributions [19, 95]. Indeed, the power law family of distributions can be shown to be asymptotically nested in the stretched exponential family, with the latter converging to the former in the limit of shrinking stretched exponential exponent [23–25]. The robust Wilks test allows you to compare power laws and stretched exponentials for any data set of interest, therefore this seemingly arcane property is highly useful in practise.

As examples, the empirical distributions of daily returns of the DJIA and SSEC indexes are displayed in the left column of Fig. 2. There are noticeable fat tails and outliers when contrasted to the inverted parabola shape expected by the Gaussian hypothesis.

Two further crucial stylized facts are the absence of long memory in returns and the prevalence of long memory in volatility, as seen in the centre and right panels of Fig. 2. The Hurst exponents may be calculated mathematically using detrended fluctuation analysis or detrending moving average analysis on the return and volatility time series.

Diagram

Description automatically generated with medium confidence

**1.3 Standard econometric models:**

The autoregressive conditional heteroskedasticity (ARCH) model [96], the generalised autoregressive conditional heteroskedasticity (GARCH) model [97], the exponential GARCH (EGARCH) model [98], the integrated GARCH (IGARCH) model [99], the fractionally integrated generalised autoregressive conditional heteroskedasticity (FIGARCH) model [100], the exponential GARCH (EGARCH) model [98], the integrated

AR(p) denotes an autoregressive model of order p.

􏰗p k=1

If a1, ap are the model parameters, a0 is the constant, and t is white noise The error component t is separated into a stochastic piece zt and a time-dependent standard deviation t so that t =tzt to represent a financial time series using an ARCH process [96]. (4) A powerful white noise method is used to generate the random variable zt. The 2t series is represented by

is given by [97]

σ2 =ω+α ǫ2

t 1 t−1

+···+α ǫ2

q t−q

yt = bxt + ǫt

ǫ t | ψ t ∼ N ( 0 , σ 2t )

+β σ2 +···+β σ2 =ω+ 1 t−1 p t−p

􏰗q i=1

αǫ2

i t−i

+

􏰗p i=1

βσ2 , (6) i t−i

r(t)=a0 +

akr(t−k)+ǫt, (3)

􏰗q

σ2=α+αǫ2 +···+αǫ2 =α+ αǫ2 , (5)

t 0 1t−1 qt−q 0 it−i i=1

whereα0 >0andαi ≥0fori>0.

A regression with coefficient b between two variables xt and yt is fitted with the GARCH(p, q) residuals model, where p is the order of the GARCH terms and q is the order of the ARCH terms. The GARCH model is often used in finance to represent the return time series as (4) with provided by (6).

In the ARCH, GARCH, and EGARCH models, the autocorrelation of volatility decays exponentially. Integral model volatility, on the other hand, has a long memory. Empirical studies show that GARCH-type models in volatility with no long memory can only partially capture the multifractal nature of financial time series, whereas other models with a long memory component can capture the apparent multifractality, which is spurious by definition in integrated models [10–11]. As a result, benchmark economic models overlook a crucial feature of market complexity: financial markets' multifractal nature [13, 14].

**1.4 Introduction to multifractality**

Multifractality has been documented in many complex systems, including financial markets [113] [115–121]. Frisch and Parisi coined the term "multifractal" in 1983 [122], and Benoit B. Mandelbrot [123], the well-known Father of Fractals [124], confirmed their work. In their studies of turbulence in fluid mechanics, Novikov [125] and Mandelbrot [126, 127] created the concept of multifractality. The Renyi entropy, on the other hand, was employed by two groups led by Grassberger and Procaccia to integrate the fractal, information, and correlation dimensions into a single statement of generalised dimensions [128–130], with the primary purpose of characterising odd attractors in nonlinear dynamics [131]. In addition to applying multifractal theories to fractal growth processes, Halsey et al. were crucial in the early development of multifractal theories [132].

The function (q) or the function f () can be used to characterise multifractals, where q is the order of particular moments, (q) is the mass exponent function, is the singularity strength, and f () is the singularity spectrum. These two representations are linked by the Legendre transform [122, 132]. as a result

and

α = dτ(q)/dq (7a) f (α) = qα − τ(q). (7b)

Concerning the τ(q) representation, there are two additional equivalent functions Dq and H(q). The Dq function presents the generalized dimensions that are determined by

while H(q) is defined by

Dq = lim τ(q′) , (8) q′→q (q′ −1)

H(q)= lim τ(q′)+1, (9) q′→q q′

The generalised Hurst exponents relate to this. The precise meaning of these variables, as well as their relationships, will be revealed later.

In the study of turbulence, the structure function approach and the partition function approach are two well-known approaches for the analysis of multifractals [122, 125–127, 133–136]. The apparent parallels between turbulence and financial markets have spurred interest in the multifractal nature of financial time series [17, 18], but the analogy has its limitations [137]. The structure function approach was prominent in the first wave of multifractal analysis in econophysics [138]. Multifractal detrended fluctuation analysis has swiftly become the main technique for analysing financial and other time series since Kantelhardt et alfoundationalwork .'s in 2002 [139]. Podobnik and Stanley proposed the detrended cross-correlation technique for non-stationary time series [140] in 2008, and one of us (Wei-Xing Zhou) adapted it to analyse multifractal time series [141]. The third wave of simultaneous multifractal analysis of two time series, as well as the introduction of a plethora of new multifractal analysis techniques, began with these significant papers. All financial activity in the financial markets revolves on the concept of risk. Large financial swings, especially financial "tsunamis" like the one that erupted in 2008 [42], are typically followed by major alterations in risk perception and regulation [40, 41]. Theoretically and practically, identifying, assessing, and forecasting financial risks is crucial in risk management. Econophysicists have showed that multifractal analysis may be used to study market risks in a new way. The behaviour of small fluctuations is described by big singularity strengths, whereas the behaviour of large fluctuations is described by small singularity strengths. As a result, there have been various attempts to quantify market inefficiencies, evaluate financial volatility, and so on using multifractal analysis.

This review goes through multifractal analysis of financial time series in great detail. We begin with different multifractal analysis methodologies for univariate and multivariate time series in Sections 2 and 3. We also look at techniques that have been established in other fields and see if they may be applied to econophysics. In Section 4, we go through several essential mathematical and econophysical models that can help us better understand how financial markets are multifractal. Section 5 examines key aspects and issues with the algorithms used in empirical multifractal studies, many of which are frequently overlooked by researchers, resulting in inaccurate conclusions. We conclude that multifractality exists in financial markets in Section 6 after a thorough assessment of the literature on empirical multifractal analysis of financial time series. The important distinction between apparent and effective multifractality, as well as the multiple reasons of apparent multifractality, are discussed in Section 7. Section 8 looks at how the multifractal nature of financial time series may be used in a variety of ways. In Section 9, we look at some of the most pressing challenges and provide recommendations for further research.

Although this research focuses on multifractal analysis of financial markets, the majority of the information may be applied to multifractal time series analysis in any area. A major section of this study may also be utilised to drive multifractal analyses of surface and higher-dimensional space measurements. Almost all works on multifractal analysis of financial time series, with a few exceptions, concentrated on individual time series. Further research may be done on an ensemble of financial time series of individual assets [165]. Ensemble averaging [166–170] revealed a subtle discrete hierarchy characterised by complex fractal dimensions [171], revealing the multifractal characteristics of Diffusion-Limited Aggregations.

###### Motivation:

The diversity of Mandelbrot set which is followed in almost every natural process

Formation of natural patterns that are a part of Mandelbrot set and correlation with chaos theory

Prediction of future events that can be derived by pattern analysis of fractals

###### Objective:

A time series model is used to create pattern analysis of a fractal geometry and performance of different financial markets

Conclusions are derived based on known patterns to predict probable outcomes of future

To make financial decisions better backed by fractal mathematics and geometry

**CHAPTER-2**

**LITERATURE SURVEY**

Wei et al. [29] proposed utilising a multifractal volatility-based econometric model and extreme value theory to calculate daily Value-at-Risk (VaR). Using high-frequency SSEC data, VaR back-testing methodologies show that the proposed VaR measures beat several linear and nonlinear GARCH-type models at high-risk levels [26]. Lee et al. used the MMAR model [37] to forecast VaR. According to an empirical analysis of daily data from the KOSPI, S&P500, and USD/KRW foreign exchange from 1990 to 2012, MMAR produces more consistent and accurate VaR forecasting than Gaussian VaR, t-distribution VaR, GARCH VaR, and historical VaR. From 5 January 2006 to 31 December 2007, Batten et al. examined the 5-minute returns of EUR/USD spot quotes and trading ticks and discovered that a modified MMAR model beats both historical simulation and the GARCH(1,1) location-scale VaR model [41].

Malo studied the Nord Pool power spot prices from March 1998 to January 2006 and showed that the Copula-MSM model can anticipate VaR better than GARCH models [31]. Liu and Lux examined daily data from two stock market indices (the Dow Jones composite 65 average index and the NIKKEI 225 average index) from January 6, 1970 to December 31, 2008, two foreign exchange rates (USD/BPG and DEM/GBP) from March 1, 1973 to December 31, 2008, and a bond portfolio of US 1- and 2-year treasury constant maturity bond rates from June 1, 1976 to December 31, 2008 [40]. In terms of forecasting, they found that the bivariate MSM model with heterogeneous volatility correlations outperforms both the homogeneous benchmark and the bivariate DCC-GARCH model [20]. On the extreme returns of six daily commodities futures markets (Brent and WTI crude oil, cocoa, cotton, copper, and gold), Herrera et al. proposed the MSM Peaks-Over-Threshold (MSM-POT) model and compared it to self-exciting models and the GARCH-EVT approach [38]. The most accurate VaR estimations were found to be provided by the MSM-POT model.

Bacry et al. investigated the effectiveness of MRW, normal, and t-Student GARCH(1,1) models in 1-day conditional VaR forecasting using the daily values of four foreign exchange rates (CAD, JPY, CHF, and GBP) against the USD [43]. With VaR values ranging from 0.5 to 10%, the MRW model gives equivalent or superior results when compared to GARCH-type models. The advantage of the MRW model over GARCH-type models in VaR forecasting was confirmed for the daily returns of five stock market indices (CAC40, FUTSEE, DAX, DJIA, and NIKKEI) between 1990 and 2005 [44].

The Markov-switching multifractal (MSM) model, which is based on the multifractal model of as-set returns (MMAR), is another significant path in multifractal volatility forecasting (see an excellent composition by Calvet and Fisher [984], and also Section 4.2 and Section 4.3). In their seminal work [45], Calvet and Fisher created a robust volatility forecasting model. Lux and Morales Arias demonstrated that the MSM model outperforms GARCH, FIGARCH, the stochastic volatility model (SV), and the long memory stochastic volatility model (LMSV) in terms of volatility predictions [32]. Lux and Kaizoji examined the performance of GARCH, FIGARCH, ARFIMA, and lognormal MSM models in volatility forecasts using daily data from two samples of stocks on the Tokyo Market from 1975 to 2001 [986]. In terms of relative mean squared error (MSE) and mean absolute error (MAE), they observed that the long memory models ARFIMA and FIGARCH outperform the short memory GARCH and ARMA models, with the log-normal MSM model performing best. The performance differences between these models become progressively significant over longer forecasting horizons. The outcome is the same for sub-periods (1986-1990, 1991-1995, and 1996-2001), as well as volume projections. From January 1, 1996, to September 2, 2013, Ben Nasr et al. examined daily data from the Global Dow Jones Islamic Market World Index and discovered that the MSM model outperforms long memory GARCH-type models (FIGARCH and FITVGARCH), which outperform the short-memory GARCH model [17].

Lux and Morales-Arias looked at daily data from 25 all-share stock indexes, 11 10-year government bond indices, and 12 real estate security indices at the nation level from January 1990 to January 2008 [42]. In terms of volatility predictions, they observed that the Binomial MSM and lognormal MSM with either normal or Student innovations perform similarly to GARCH and FIGARCH. They also observed that combined predictors outperform solo models, with the FIGARCH- MSM model having the best performance. Chuang et al. observed that the MSM model performed better than implied, GARCH, and historical volatilities in realised volatility forecasting than implied, GARCH, and historical volatilities in daily data of the S&P 100 index option and equity option from 3 January 2000 to 31 October 2009 [38]. Lux et al. [990] validated this conclusion by using daily WTI oil price data from 2 January 1875 to 31 December 1895 and from 2 January 1985 to 24 March 2014. Segnon et al. verified that the MSM model beat GARCH and FIGARCH on daily carbon dioxide emission allowance pricing from 16 January 2009 to 20 January 2015 [19]. Liu et al. examined Binomial MSM models with normal and Student-t innovations against GARCH-type models (GARCHN, GARCH-t, and GARCH- Skewed t) using daily closing prices of SSEC from 15 July 1991 to 8 June 2015 and found that the Binomial MSM model with Skewed-t innovations prevailed [43].

During the past 40 years, the emergence of insurance market crises has been connected to declining GDP (MAPFRE Economics Review, n.d.), and GDP dependency may be found in both established and emerging nations. At the same time, the features of the insurance market's growth may have an influence on the broader economy of the country. A issue in the insurance business might set off a financial disaster. For example, in 2008, the United States spent 182 billion dollars to preserve the well-known insurance company American International Group (OECD, 2020), whose debt holders included international investment funds.

Significant financial losses may have been prevented if the preconditions for the crisis had been detected. The insurance industry has been reorganised rather than collapsing as a result of the current economic crisis. Certain insurance products have grown in popularity, but their risks have increased dramatically. For example, in 2008, the United States spent 182 billion dollars to preserve the well-known insurance company American International Group (OECD, 2020), whose debt holders included international investment funds.

Significant financial losses may have been prevented if the preconditions for the crisis had been detected. The insurance industry has been reorganised rather than collapsing as a result of the current economic crisis. Certain insurance products have grown in popularity, but their risks have increased dramatically. Risk causation and new behavioural patterns As a result of these and other changes, insurers may need to change their primary business model. However, they cause market asymmetry and instability, making it more difficult to predict the market's state.

Global trends in the insurance market have been steadily increasing over the previous 10 years. The onset of the COVID-19 problem did not result in a market meltdown in most nations. Insurers' earnings and capital positions were not as strong in 2020 as they had been in previous years, but the market environment remained stable in general (Ogilvi, 2021). In 2019–2020, gross insurance premiums grew in both life and non-life insurance. Gross payments increased as well, with growth rates ranging from 80.2 percent in Russia to 11.7 percent in Malaysia, although insurance activity did not decline internationally. It should be mentioned that the insurance business has been dominated by positive trends in dynamics since 2001. (Kozmenko et al., 2009, pp. 51-52).

Despite the economy's overall downward trend, a wide examination of the dynamics of the Ukrainian insurance industry demonstrates its growth and concentration. As a result, the number of insurance firms decreases as premiums, payments, and contract numbers grow. The number of registered insurers decreased from 404 in the first quarter of 2014 to 210 in the fourth quarter of 2020, while the number of life insurance firms decreased from 61 to 20. Nonetheless, the insurance business in Ukraine is not as concentrated as, instance, the insurance market in the United States, where only 22 corporations provide all types of insurance services (Amadeo, 2019). In prior times (Kozmenko et al., 2009, p. 33), the number of insurance businesses in the European Union decreased by 775 from 2000 to 2007. In general, the insurance market in Ukraine is developing in the same way as markets in emerging countries are developing. First and foremost, it concerns market development issues (Polinkevych & Kamiski, 2020, p. 22), such as economic instability in the country, insufficient state control over market status and state regulation of service pricing, ineffective demand, a lack of insurance culture, and so on (Polinkevych & Kamiski, 2020, p. 22). As in the past, a decline in GDP did not result in a significant drop in insurance premiums (MAPFRE Economics Review, n.d.). The experience of developing economies (Mexico and Brazil) demonstrates that expansionary monetary policy has a positive impact on insurance market dynamics. At the same time, these countries' use of lower interest rates has intensified financial market volatility, which might harm the insurance business.

Market risk may be predicted and managed in a variety of ways. Fundamental analysis is the oldest and perhaps the most fundamental financial market methodology. In the context of a potential investment or transaction, this comprises investigating a firm, industry, or market, as well as the economy, in order to understand the source of stock price swings. The results are then used to try to predict how a stock will move in the future. In this type of fundamental investigation, an underlying rationale is postulated. It is anticipated that the price, bond, derivative, or commodity will change as a consequence of some event or fact, which is virtually always produced by another external event [4]. The essential idea of this method is that if the underlying reason can be discovered early enough, the event may be foreseen and proper risk or investment management can be implemented. In the real world, however, causes are usually opaque or undetectable. Critical information is frequently unknown, indeterminable, buried, or even altered (see the Enron or Parmalat corporate scandals, for example)[25]. Furthermore, some or perhaps all market participants may misunderstand information.

Major brokerage companies continue to recruit a large number of fundamental analysts to spot obvious, frequently right trends. For example, certain currency rates may approach a fundamentally defined level before reversing or continuing upwards. However, on an open market, this type of analysis is severely limited. The mood, or process, that connects news to pricing is sometimes ambiguous and open to personal interpretation. In hindsight, fundamental analysis may frequently be recreated to produce precise event projections. Before the fact, however, two diametrically opposed outcomes might have been equally plausible[5]. As a result, while authenticity is sometimes apparent, it is not necessarily the best foundation on which to build a risk management system. As a result, the financial sector has produced and continues to develop new forms of data analysis that are more quantitative in nature. The second oldest sort of analysis is technical analysis. This includes detecting patterns (real or imagined) as well as analysing price, volume, and indicator charts to determine whether to purchase or sell[24]. After falling out of favour in the 1980s, the field resurfaced in the 1990s when the general public began to trade stocks and stocks through the Internet.

##### The advent of this communication technology gave rise to "modern finance theory," which includes analytical methodologies based on accidental mathematics, probability theory, statistical analysis, and probabilistic field modelling. The key idea is that while we cannot predict the exact amount of a future price, we can estimate its (short-term) fluctuation unless the statistical features of the fluctuation change with time[6]. It signifies that you have the ability to finish the assignment. In this perspective, risk should be considered as a measurable quantity that can be managed. Modern market analysis is based on this idea. FMH is inevitably induced by the non-ergodicity of financial time series. H. They're inaccurate statistically since they're random fields.

##### CHAPTER-3

**MULTI FRACTAL ANALYSIS**

###### Partition function approach

Consider a measure m contained in a geometric support F whose density at point t′ is m (t′). t′F m (t′)dt′ = 1 by definition. m (t′)dt′ is the measure in the vicinity of t′. We cover the geometric support F using boxes of size s, based on the notion of the standard box-counting approach [24]. In the tth box B(s, t), the integrated measure m(s, t) is

*m*(*s*, *t*) = *m* ̇ (*t*′)*dt*′, (10) *t*′∈B(*s*,*t*)

where 􏰕*t m* ̇ (*s*, *t*) = 1 for any *s*. The fractal dimension of F can􏰕be determined as follow

ln *t*[*m*(*s*,*t*)]0  
*Df* :=*D*0 =lim , (11)

t[m(s, t)]0 denotes the number of non-empty boxes required to cover the support F. It's worth noting that the fractal dimension is also known as the capacity dimension or similarity dimension. It is self-evident that a box should not be counted if there is no measure dis- tributed in it. When dealing with financial time series, D0 = 1 is the most common setting. We plot t[m(s, t)]0 against s on log-log scales in order to estimate D0, and the slope of the linear part in an appropriate scaling range is used.

The measure's information entropy, also known as Shannon entropy, is [142, 143].

*I*(*s*) = − 􏰗 *m*(*s*, *t*) ln[*m*(*s*, *t*)], (12)

*t*and one can define the information dimension *D*1 as follows

σ=lim *I*(*s*) , (13) *s*→0 ln(1/*s*)

which was introduced by Balatoni and Re ́nyi in 1956 [144, 145] and used to study strange attractors [146, 147].  
A third index used to characterize the measure is the correlation dimension [148, 149].

The correlation dimension

10

s→0 ln 1/s

was originally introduced to study the scaling behavior of the correlation integral of time series of length N:

1􏰗N 􏰗2

C(s)= lim 2 H(s−|⃗xi −⃗xj|)≈ [m(s,t)] , (14)

N→∞N i,j=1 t

where H(x) is the Heaviside function. The correlation dim􏰕ension is defined as follows

ln t[m(s,t)]2

ν = lim . (15)

s→0 ln 1/s

These three dimensions D f , σ and ν can be unified into one framework of generalized dimensions [128–130]. We

define the q-order Re ́nyi entropy as [150, 151]

Iq(s) = lim 1

ln 􏰗[m(s, t]p, t

(16)

(17)

(18)

and the generalized dimension as [152]

where

 1 

Iq(s) Dq = lim

1 lnχ(p,s)

lns

 lim 

, m(s, t) ln[m(s, t)]

ln s

q 􏰥 1 , q=1

p→q 1 − p

l n χ ( p , s ) s→0p−1 lns

s→0 ln(1/s)

= lim lim

s→0 p→q p−1

=

 􏰕  t lim

χ(q, s) =

􏰗

t

 s→0 q

is the measure's q-order partition function. It's simple to check that D0 = D f, D1 =, and D2 = v. In reality, we may use Eq to calculate Dq (17). To derive the slope Dq for a given q, we plot Iq(s) against ln s for various box sizes s and do linear regression in a reasonable scaling range.

Eq. (17) may be rewritten as follows: or

(q−1)Dq =limlnχ(q,s) (19) s→0 ln s

χ(q, s) ∼ sτ(q), (20) τ(q) = (q − 1)Dq. (21)

[m(s, t)]

In practise, we may use Eq to compute (q) (20). We can run a linear regression of ln (q, s) versus ln s in a tolerable scaling range to get (q, s) for various box sizes s for a given q. (q). Eq, on the other hand, may be used to convert (q) from Dq (21).

Xiong and Shang recently proposed and showed the link between the related multifractal functions using a variance-weighted partition function approach [53]. In numerical simulations, the variance-weighted partition function strategy surpasses the normal partition function approach. Based on the box-counting principle, the geometric support is split into non-overlapping boxes of size s. The following relationships [32] define the intensity of local singularities in the tth box B(s, t):

m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s,t) m(s

Ns is the number of boxes in which the singularity strengths are contained [, + d) (). As a consequence, Ns() = (′)sf(′)d′ (23) is the set's fractal dimension, where () is the density of the singularity strength and f () is the fractal dimension of the boxes under consideration [32], which is also known as multifractal spectrum or singularity spectrum.

One may deduce Eq. (22) and f () from Eq. (22) using local and pointwise singularity approaches [154–156]. (23).

We get (q,s)= sq′(′)sf(′)d′ = (′)sq′f(′)d′ by inserting Eq. (22) into the partition function (18) and rewriting the sum into an integral over. (24)

Assume that () is non-zero and non-singular. As a result, the integral's leading behaviour has a constant proportional component. Because s is small, the value of (q) that minimises the power q′ f(′) will dominate the integral, according to the steepest descent approach. As a result, we replace ′ with (q), which is defined as follows:

and

Then, Eq. (24) becomes

It follows from Eq. (25) that

and

d􏰂qα′ − f(α′)􏰃/dα′􏰆􏰆􏰆α′=α(q) = 0 d2 􏰂qα′ − f(α′)􏰃/dα′2􏰆􏰆􏰆α′=α(q) > 0.

qα− f (α) χ(q, s) ∼ s .

f′(α) = df(α)/dα = q f′′(α) < 0,

(25a) (25b)

(26) (27a) (27b)

(28)

(29)

(30a)

which means that the multifractal spectrum f (α) is a concave function and its slope at point (α(q), f (α(q)) is q.

Comparing Eq. (20) and Eq. (26), we have

τ(q) = qα − f (α). Taking the derivative of Eq. (28) with respect to q, we have

dτ(q) dα d f (α) dα = α + q −

= α.

Rewriting Eq. (28) and Eq. (29), we have

and

dq dqdαdq α = dτ(q)/dq

f (α) = qα − τ(q) .

numerically α(q) using Eq. (30a) and f (α) using Eq. (30b). To qualify a multifractal measure, we can use either (q, τ)

or (α, f ), which are equivalent.

The singularity strength α(q) and its spectrum f (q) can be computed by linear regressions in log-log scales using the

following equations:

and

α(q) = lim s→0

􏰕t μ(q, s, t) ln[m(s, t)] ln s

,

(32a)

(32b)

􏰕t μ(q, s, t) ln 􏰂μ(q, s, t)􏰃 f(q) 􏰢 f(α(q)) = lim .

s→0 ln s

Plot t(q, s, t) ln [m(s, t)] vs ln s to get the scaling range, then apply linear regression to determine the slope. The f (q) function can alternatively be found using Eq (32b). By combining Eqs. (20) and (28) we can easily show Eq. (28) (32). The mass exponent function (q) may be calculated using Eq (28).

For financial volatilities, we can use the inverse partition function [159]. To show the partition function approach, we start with a high-frequency volatility time series.

The price time series of a financial asset is denoted by I(t): t = 1, T. At the finest temporal resolution, the logarithmic return r(t) is determined.

r(t) = ln I(t) − ln I(t − 1), (33) where t = 2, · · · , T . The absolute return is utilized as a proxy for volatility according to

v(t) = |r(t)|. (34)

Figure 3(a) shows a slice of the S&amp;P 500 index's 1-min volatility time series using the partition function technique, in which the original volatility time series is partitioned into N boxes of equal size s = T/N. The box sizes are selected so that the number of boxes of each size is an integer to cover the whole time series.

Chart, line chart

Description automatically generated

We define the continuous volatility measure u(t′) for any t′ ∈ (t − 1, t] by

u(t′)=􏰕v(t) , (35)

Tt=1 v(t)

where t = 1, · · · , T . The cumulative function of the volatility measure is obtained as

􏰙t 0

Figure 3(a) shows the function U(t) of v(t). Figure 3(b) shows the function U(t) of v(t) (b). n(s) = U(ns) U(ns s), (37) which is also shown in Fig. 3 determines the measure in each box of size s. (b). To eliminate edge effects, one usually computes the measure in a discrete manner, n s t=(n1)s+1, where T/s must be an integer (see Section 5.1.2 for more information). When using the continuous measure u(t), there are more options for s values since N can be any integer.

The direct partition function q(s) can be approximated for a given order q by using

n=1

U(t) =

u(t′)dt′. (36)

μn(s) =

u(t), (38)

χq(s) =

􏰗N

q

In Fig. 3, we show [q(s)]1/q1 as a function of s for various q's (d). The scaling exponent function is derived using power-law regressions applied to data points in the scaling range, which spans three orders of magnitude (q). Fig. 3(f), which indicates that (q) is a nonlinear function of q, confirms the presence of multifractality in the volatility measure.

Now we'll look at the inverse partition function. A series of exit times sj(): j = 1, J may be found sequentially for each threshold by J.

sj =inf{t:U(t)􏰤 jμ}, (40) j=1

where J = 1/μ is an integer. Graphically, Fig. 3(c) shows how the exit times sj can be determined. The inverse measure is defined as the normalized exit time

μ †j ( μ ) = s j / T , ( 4 1 ) and the inverse partition function can be determined as follows:

χ†(μ,p)=􏰗J 􏰛μ†j(μ)􏰜p, (42) j=1

Figure 3(e) shows the dependence of [χ†p(∆v)]1/(p−1) as a function of the thresholds ∆v for different p values. Power- law scaling can be observed over about three orders of magnitude:

† τ†(p)

χp(μ) ∼ μ . (43)

Applying power-law regressions to the data points in the scaling range, as illustrated, yields the scaling exponent function (p).

The link between (q) and (r) is an enthralling topic (p). For both discontinuous and continuous multifractal data, Mandelbrot and Riedi analytically demonstrated an elegant inversion formula [60, 61]. Roux and Jensen [62] separately identified the precise relationship between the direct and inverse scaling exponents for classical binomial measurements. The inversion formula for multinomial measures was given a simple "proof" by Xu et al. [63], which is explained here.

Let be a probability measure with the integral function M(t) = on the range [0, 1]. ([0, t]). Then inft:M(t)>s may be used to establish its inverse measure, ifs1

μ†=M†(s)= 1, ifs=1 , (44) † 􏰕 􏰕n −1

M (s) denotes the inverse function of M. (t). If is self-similar, then = i=1 pi(mi ()) holds, where mi's are similarity maps with scale contraction ratios ri (0, 1) and ni=1 pi = 1 with pi > 0. The Legendre transform f () of is the multifractal spectrum of measure, which is defined by the generating function dq d (p) (p) We get n by combining Eq. (47), Eq. (49), and the Legendre transform.

ii

pqr−τ = 1.

It can be shown [60, 61] that the inverse measure μ† is also self-similar with ratio ri† = pi and p†i = ri, whose

multifractal spectrum f†(α†) is the Legendre transform of τ†, which is defined implicitly by

􏰗n

(p†i )p(ri†)−τ† = 1. (46)

i=1

It is easy to verify that the following inversion formula holds

􏰑τ(q) =−p

τ†(p) = −q . (47)

Two equivalent testable formulae follow immediately

􏰑 τ(q) = −τ†[−1](−q) , (48)

τ†(p) = −τ[−1](−p)

The superscript [1] denotes the inverse function operator. Figure 3(f) illustrates that the inversion formulae (48) hold for the high-frequency volatility time series of the S&amp;P 500 index [59].

It is possible to correlate the direct and inverse singularity strengths, as well as the direct and inverse singularity spectra. Eq. (47) is derived from Eq .i=1

dτ(q) −dp 1

α(q)= = † =† . (49)

(45)

f(α)=qα−τ=−τ†(p)α+p=α[−τ†(p)+pα†]= f†(α†)/α† =αf†(1/α). (50) Refs. [60, 61] also provide thorough verification of these results. The relationship between direct and inverse generalised dimensions is simple to find [64]:

􏰑 D†(p) = q/[1 + (q − 1)D(q)]

p = −(q−1)D(q) , (51) where D†(p) is the inverse generalized dimension function.

Almost all work on multifractal analysis of financial time series has focused on individual time series, with a few exceptions. Further research may be done on an ensemble of financial time series of individual assets [65]. Ensemble averaging [66–70] revealed a subtle discrete hierarchy characterised by complex fractal dimensions [71], revealing the multifractal characteristics of Diffusion-Limited Aggregations.

The mass exponents for quenched and annealed materials, quen(q) and ann(q), are as follows:

􏰓ln χq(s)􏰔 ∼ −τquen(q) ln s, (52)

ln 􏰓χq(s)􏰔 ∼ −τann(q) ln s, (53)

The angle brackets represent the ensemble average of all time series. The quenched exponents are more representative of normal ensemble members, but the annealed exponents are more sensitive to rare samples of the ensemble with unusual q values (s).

The concept underlying ensemble averaging is that the dynamics of a "group" of financial instruments are monitored many times. To evaluate market risks from individual equities, the ensemble averaging technique is an alternative to utilising the market index. It's also not the same as averaging the risk of a big number of different equities. Furthermore, while the method was created with the partition function in mind, it is easy to adapt to various multifractal analytic methods.

###### Structure function approach:

Another important statistical variable in turbulent environments is the structure function of velocity increments [33, 34]. Traditional methodologies were used to measure the structure function and its nonlinear scaling properties [35]. Nonlinear scaling behaviour is described by the term multifractality [22, 36]. The structure function approach has also been used to study financial time series [18, 38, 73]. The qth-order structure function is sometimes referred to as the qth-order height-height correlation function in the literature [74, 75]. Consider the following time series X(i): I = 1, N. Over time scales, increments or inventions are characterised as

∆X(i, s) = X(i) − X(i − s).

The logarithmic price of financial assets is X(i), and the return is X over a time interval of s. The increment distribution's qth-order moment is defined as the qth-order structure function.

We emphasise that q 0 for time series in turbulence [135], finance [138, 176], and other domains since X(i, s) may be zero and the moments of negative orders are not given. When q = 0, we obtain K(0, s) 1, which is independent of the scale s. The autocorrelation function X(i)X(i s) is proportional to K(2, s) for q = 2 [38, 76].

For self-similar time series, we expect to have

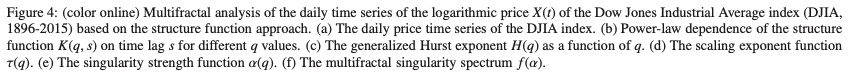
K(q, s) ∼ sζ(q) 􏰢 sqH(q),

A substantial body of research has proven the gain-loss asymmetry [205, 207–215], indicating that the optimal investment horizon is longer for profits than for losses. Several models have been created to replicate the gain-loss imbalance [216–218]. Despite this, certain assets [202, 219] do not have a significant gain-loss asymmetry.

According to a frequency space study based on the discrete wavelet transform [218], the gain-loss asymmetry is largely produced by the low-frequency content of the price series, and the asymmetry disappears after enough of the low-frequency material is removed. Gain-loss asymmetry is instead revealed to be caused by non-Pearson-type autocorrelations in the time series [220]. Furthermore, the gain-loss imbalance is reduced when the temporal dependence structure is abolished by shuffling the time series [221]. The gain-loss imbalance can be connected to the leverage effect in the following way. Remember that the leverage effect happens when volatility increases abruptly after a large negative return and then reduces exponentially again [222–224]. On the other side, there is no obvious difference in volatility after a positive return. And rather than the other way around, the causality is from (negative) returns to future volatility, in the sense that a change in volatility has no influence on the predicted return in the future. The leverage effect is the impact of a loss on a firm's risk perception, which grows when the equity over debt ratio declines as a result of the loss [222]. It also depicts an investor's reaction to a loss, in which they rush to assess their risk exposure and rebalance their portfolios, resulting in massive price fluctuations. The leverage effect can be used to explain I the asymmetry in the average time it takes for a fixed percentage price drop versus a fixed percentage price gain (for example, 10 days for a loss of 5% and 20 days for a gain of 5% on the DJIA) and (ii) the fact that this asymmetry in waiting times for positive vs negative objectives is stronger for indices and weaker or absent for individual stocks [218, 221]. ? Consider a -5 percent drop as a firm target. To achieve this, a sequence of daily results that are more negative than positive is employed. When a negative loss occurs on one day, the leverage effect causes the amplitude of the next day's return to increase. Because we are computing a waiting time conditional on a cumulative drop of -5 percent, the future returns will tend to be negative and with greater amplitude due to the leverage effect. The leverage impact will be weaker or absent if there are no or weak negative returns along the price path, and the waiting time to achieve a positive gain of +5% will be positive for the majority of daily moves, and the leverage effect will be weaker or absent. As a result, the average waiting time to achieve a positive return (+5%) target is longer than for a negative return goal, since the amplitude of daily returns for a series of daily returns included in a positive level (+5%) tends to be small. The asymmetry of a single stock is minimal since leverage has a short memory and the difference between the number of up and down daily moves necessary to satisfy the target threshold is not large. Correlations between the returns of the stocks that make up the portfolio must be considered in the case of an index, or equivalently a portfolio of N stocks. Because the leverage effect is magnified by averaging over the idiosyncratic residuals of the constituent stocks, the gain-loss imbalance is amplified by these correlations, which are frequently positive (most stocks move closely together on average). This also implies that the greater the average correlation coefficients of a portfolio's component stocks are, the larger the gain-loss asymmetry is.

Figure 4 shows the findings of a structure function-based multifractal analysis of the daily time series of the logarithmic price X(t) of the Dow Jones Industrial Average index (DJIA, 1896-2015).

Graphical user interface

Description automatically generated

**Chart, line chart

Description automatically generated**

###### Wavelet transform approach:

The process of classifying a large number of items into distinct categories is known as classification.

The wavelet transform is a mathematical technique for analysing temporal data. It may be used to investigate the specific structure of fractal and multifractal time series. The function X(twavelet )'s transform is

1􏰙 +∞ 􏰍t−b􏰎

Cψ(b, s) = s X(t)ψ s dt, −∞

where b ∈ R is the position parameter, a ∈ R+ is the dilation parameter, ψ(t) is the analyzing “mother” wavelet. Gaussian wavelets are widely adopted [241] and are defined as derivatives of the Gaussian function:

dn 2

gn(t) = g(n)(t) = cn dtn e−t /2,

The normalisation coefficient is cn. The Mexican hat is a Gaussian wavelet of second order (n = 2). To execute a wavelet transform on a time series X(i)N, discretize Eq., where b can take any i=1 value throughout the sample spacing and s takes ns scales to construct a geometric sequence:

sj =λj−1smin, 1􏰣 j􏰣ns,

The wavelet transform is used to identify several types of singularities in signals on the time-scale plane, depending on the order n of the mother wavelet's derivative (78). The wavelet transform ignores polynomial dependence up to order m and only identifies higher order powers if the mother wavelet satisfies xm+1(s)dx = 0. When m = 0, for example, the wavelet transform is unaffected by the signal's level and focuses on the local slope. The wavelet transform offers information on the signal's local curvature and is independent of the signal's level and local slope when m = 1. It's also worth mentioning that the Hurst index of a long-range correlated time series may be calculated directly using orthogonal wavelet transforms and biorthogonal wavelet transforms [24–27].

A more elegant approach for reducing redundancy in continuous wavelet transformations is the wavelet transform modulus maxima (WTMM) [25, 24–31]. At each scale sj, there exist several local maxima of |W(sj,i)|. These maxima form several maxima lines in the (i,s) plane. In a wide variety of time series, the WTMM can detect all singularities [22]. When dealing with the q 0 circumstance, the traditional partition function technique may fail. We will demonstrate how multifractal analysis based on WTMM can overcome this problem since the wavelet coefficients on the WTMM line have non 0 local maxima [51].

###### Detrended fluctuation approach:

Long-range relationships in coding and noncoding DNA nucleotide sequences were initially investigated using the Detrended Fluctuation Analysis (DFA) [68]. Shortly after, MF-DFA, a multifractal extension of DFA, was produced [69, 70]. Since the work of Kantelhardt et al. [39], MF-DFA has become one of the most important methodologies for multifractal analysis in a number of areas.

In the MF-DFA technique, the trend function is a polynomial [39], and the linear trend is the most usually utilised.

􏰚 􏰗l

X(i) = akik, l = 1,2,··· . (16)

k=0

The detrending process for l = 1 is shown in Figure 6. The data points at each end of the time series should not be included to construct a box in the case of Ns N/s, as illustrated in the figure. Otherwise, as will be described, when fmax = f (q = 0) > 1, incorrect results are produced. In the traditional method for applying MF-DFA, researchers remove the constant shift X from the increments X and focus on the cumulative profile.

i j=1

The sample mean of the X(i) series is X, while the sample mean of the X(i) series is Y. However, when the trend function is polynomial, this change has no impact on the results.

In the MF-DFA technique, the choice of polynomial order is a subtle matter. Oswiecimka et al. discovered that the calculated singularity spectra can be very sensitive to the order of the detrending polynomial using classic mathematical models (fractional Brownian motion, Levy process, and binomial measure) and real-world time series (foreign exchange rate and literary texts) [27]. Without a good understanding of the underlying mechanism that drives the dynamics, it's impossible to identify which polynomial order to use. In any case, we may check if the Hurst index H(2) for financial time series like returns is close to 0.5, based on the underlying stylized fact of the absence of long memory in financial returns [60], which may not hold at the transaction level [22].

###### Other Methods:

###### 3.5.1 Multiplier Method:

The term "multiplier" was used by Novikov to describe the intermittency and scale self-similarity in turbulent flows [25]. In econophysics, this technique might be used to investigate volatility multipliers. For high-frequency financial data, we first establish an additive measure in the time period [t1, t2], which is the total of absolute returns:

􏰗t2 t=t1

The absolute return is essentially a measure of volatility [t1, t2] [39]. The volatility time series |r(t)| is divided into boxes of the same size s. Each mother box is subdivided into daughter boxes, the first of which is known as the base. The multiplier ma,s is determined by the ratio of the measure on a daughter box to that on her mother box [30, 31].

If the multiplier is scale invariant for any two bases a and b, the Mellin transform M connects the multiplier density functions pa(ma) and pb(mb):

which leads to

[M{pa(ma)}]1/lna =[M{pb(mb)}]1/lnb,

ln 􏰖01 mqa pa(ma)dma ln 􏰖01 mqb pb(mb)dmb lna = lnb .

The mass scaling exponent τ(q) of the moment of ma can be obtained as [310, 311]

ln⟨mqa ⟩ lna

μ([t1, t2]) =

|r(t)|,

Where D0 is the fractal dimension of the measure's support We have D0 = 1 for time series.

The Legendre transform connects the local singularity exponent and its spectrum f () to (q). As a consequence,

⟨ m qa l n m a ⟩

α(q) = − q

⟨ma⟩ ln a

The scale-invariant multiplier distribution [30, 31] is regarded to be more fundamental than the traditional multifractal spectrum f. (). Furthermore, it is more accurate and requires a fraction of the time to compute than the partition function approach [30, 31]. Using 1-min high-frequency data from the S&amp;P 500 index, Jiang and Zhou discovered that the volatility multiplier is scale invariant and the volatility possesses the feature of multifractality [312]. Niu and Wang confirmed the presence of scale invariance in the multiplier distributions in the Shanghai SSEC index using simulated data from the "voter interaction dynamic system" model [31].

**3.5. 2 Multifractal Hilber-Huangspectral analysis**

The Hilbert-Huang transform [294, 315] combines the EMD and Hilbert spectral analysis, with the Hilbert transform applied to each component Ck I in Eq (123). The general trend may be summarised as follows:

􏰗n X(i) = X(i) −

􏰍􏰙􏰎

Ak(i) exp j ωk(l)dl ,

􏰗n k=1

H(ω, i) = A2(ω, i),

The Hilbert-Huang marginal spectrum of the original time series is then written as

h(ω)=􏰙 H(ω,i)di,

􏰚

Ck(i) = X(i) −

(132)

(133)

(134)

k=1

where j2 equals 1. The Hilbert-Huang spectrum is defined as the squared amplitude of each mode.

This corresponds to an energy density that varies with frequency. This method is known as EMD-based Hilbert spectral analysis (EMD-HSA), but we call it Hilbert-Huang spectral analysis (HHSA) for short.

Huang, Schmitt, Lu, and Liu established the multifractal Hilbert-Huang spectrum analysis (MF-HHSA) by generalising the Hilbert-Huang transform from second order to arbitrary order [316, 317]. The Hilbert-Huang spectrum H(, I) represents the original signal at the local level, and it may be used to generate the joint probability density function p(, A) of the frequency k and amplitude Ak. Eq. (134)'s Hilbert-Huang marginal spectrum can be written as

∞

h(ω) = p(ω, A)A2dA, (135)

0

which corresponds to the second-order moment. Huang et al. generalize Eq. (135) into arbitrary-order moments

􏰙∞ 0

Lq(ω) =

p(ω,A)AqdA,

where q ≥ 0 and h(ω) = L2(ω). In the inertial range, we assume the following scaling relation,

Lq(ω) ∼ ω−τ(q), (137)

In the amplitude-frequency space, (q) is the equivalent scaling exponent function.

The MF-HHSA technique was utilised by Li and Huang to confirm the presence of multifractality in the CSI 300 index of the Chinese stock market.

**3.5.3 EMD-based multifractal analysis**

EMD-DAMF [319] is an EMD-based dominant amplitude multifractal formalism proposed by Welter and Esquef:

Step 1: To acquire K IMFs Ck, apply EMD to X(i) (i).

Step 2: For Ck, determine the amplitude ak(i) and the timescale characteristic s. (i). The characteristic timescale s in each mode may be estimated using Hilbert spectral analysis (HSA) as the reciprocal of the mean frequency, or simply by multiplying the mean time gap over all successive zero crossings by two.

Step 3: Determine the principal amplitudes' amplitude coefficients.

uk,j := sup{max[|ak′(i ∈ Ik,j)|]}, k = 1,2,··· ,K and j = 1,2,··· ,nk,

k′􏰣k

where nk is the number of ak I local maxima, and Ik, j is a temporal support around the ak I jth maximum (i).

4th Step: Determine the qth order moment for various q values:

n k n n n n n n n

Mq(sk) = n (uk,j)q, k = 1,2,··· ,K,

k i=1

HSA in each mode can derive the characteristic time scale sk as the reciprocal of the mean (linear) frequency or simply by the mean time interval among all adjacent zero crossings.

Step 5: Determine the scalability function.

τ(q)+1−q

Mq(sk) ∼ sk .

In real-world applications, Welter and Esquef recommend setting the number of siftings to ten (K = 10) and utilising cubic spline interpolation to construct signal envelopes [39]. The technique was tested using fractional Brownian movements [32], Levy processes [31], Multifractal random wavelet cascades [32], and multifractal multiplicative cascades from the p model [34]. For larger scale modes, the EMD-DAMF technique has a limited number of scales and a small number of magnitude maxima. The latter disadvantage may have an impact on determining the scaling range.

The EMD-DFA technique [35] was proposed by Zhou et al. as an alternative. The integrated time series X is divided into intrinsic mode functions and a residual or trend component (i). To identify all intrinsic time scales s, the number of data points between each surrounding local minima is computed for each intrinsic mode function (IMF). For a given s, all IMFs are resampled by replacing all data points with 0's corresponding to scales not equal to s, and the fluctuation time series Xs(i) is formed by summing all resampled IMFs. The overall variance at scale s is represented by the root mean square of Xs. I

**3.5.4 Multiscale multifractal analysis**

Gieraltowski et al. [36] developed the multiscale multifractal analysis (MMA) based on the MF-DFA technique. They continued to estimate the local generalised Hurst exponents H(q, s) in moving windows across multiple scales after getting all values of the overall fluctuations Fq(s) using the MF-DFA technique, rather than estimating H(q) from Eq. (12) in the scaling range. This method includes two free parameters: the adjustable window size and the step. Although the MMA method was developed for MF-DFA, it may easily be used to other multifractal analysis techniques.

The MMA method was used to investigate the multifractal nature of daily returns of the Dow Jones Industrial Average (DJIA), New York Stock Exchange Index (NYSE), S&P 500 Index, Heng Seng Index (HSI), Shanghai Stock Exchange Composite Index, and Shenzhen Stock Exchange Component Index (SZCI) and nine stock market indices (CAC 40, DAX, FTSE 100, HSI, and SZCI) and nine stock market indices (CAC 40 More broadly, this procedure is similar to calculating the Fq(s) functions' local logarithmic slopes: h(q,s)= lim dlnFq(s′) (42)

s′→s dlns′

It's extensively employed in turbulence structure functions research [26, 29], as well as DFA research [330]. Estimating local logarithmic slopes is highly useful when the scaling range is constrained, and it may be used to calculate scaling ranges. One of the numerical ways that may be used to estimate numerically the local logarithmic slopes H is the linear fitting in a moving window given by the MMA methodology (q, s). Financial time series, such as the WTI crude oil market [31] and stock market indices, have been empirically examined without the term "multiscale multifractal analysis" being mentioned.

**3.5.5 Time-varying multifractal analysis**

To quantify the evolution of a multifractal spectrum, a multifractal analysis of time series in moving windows or rolling windows can be utilised. We wish to emphasise that utilising small window widths affects the accuracy of multifractal value estimates and raises the uncertainty of the results. Time-varying multifractal values have practical implications, as discussed in Section 8.2.4.

Xiong and Zhang proposed the time-varying multifractal spectrum distribution (TM-MFSD) [333] as an alternative. The fundamental idea is to look at the instantaneous autocorrelation function using multifractal analysis.

*r*(*i*, δ) = E[*X*∗(*i* − ∆/2)*X*(*i* + ∆/2)],

The window function is the time-delayed conjugation X(i) of the investigated series X, rather than the raw time series X(i) (i). WTMM [333], wavelet leaders [334, 335], MF-DMA [336], and MF-DFA [337] are examples of multifractal analysis methodologies that may be used to estimate the time-varying multifractal spectrum distribution of a time series. These methods are used to do multifractal analysis on moving windows.

**3.5.6 Phase space reconstruction**

The method of precisely embedding the time series into a phase space and calculating the generalised dimensions Dq of the data points in the phase space is a classic nonlinear dynamics method [128, 148, 149, 338], although it is seldom utilised in econophysics. The delay embedding technique [339] may be used to convert the time series X(i) into a sequence of vectors.

→−X(j)=[X(j),X(j+τ),···,X(j+(d−1)τ)], j=1,···,Nv,

where Nv is the number of points (or vectors) in the phase space, is a suitable time delay, and d denotes the phase space's embedding dimension. Different methods may be used to determine the embedding dimension d and the delay.

The number of data points within a distance R from point I is computed using the formula ci(s)=Nv j=1,jiH sX(i)X(j), (145), where H is the Heaviside function. The generalised correlation integral Cq(s), which is a generalisation of the correlation integral [28, 48, 49], is [45], (46). [38] provides the generalised dimensions.

logCq(s)

Dq ≡ lim .

Lee studied the multifractal aspects of the 1-minute volatility of the Korea composite stock price index KOSPI [46] from March 20, 1992 to February 28, 2007. The volatility time series is first detrended with a wavelet filter, and then a multifractal analysis in the phase space with = 1 and d = 1 is performed to validate the presence of multifractality.

**3.5.7 Asymmetric multifractal analysis**

Based on the stylized premise that the correlation between two assets is typically asymmetric [347–350], Alvarez- Ramirez et al. designed the asymmetric multifractal detrended fluctuation analysis (A-MF-DFA) to detect asymmetric multifractal scaling in individual time series [351]. In this method, the trend function is linear, with ai = 0 for all I > 1 in Eq. The basic idea is to create upwards (downwards) fluctuation functions for erased linear trends with non-negative (negative) slopes. The entire detrended qth-order rising fluctuation is determined analytically as

F (s)= F(s)  =⟨[F(s)]⟩| ,

In the original A-MF-DFA approach v [35], the mean absolute value of the residuals in each segment is employed. In empirical studies using the A-MF-DFA method, the asymmetric multifractal behaviour was investigated and confirmed in the daily returns of the DJIA index from 23 May 1980 to 25 August 2008 [32], the WTI oil price from 2 January 1986 to 14 December 2010 [32], the SSEC index from 19 December 1990 to 27 April 2012 and the SZCI index from 2 April 1991 to 27 April 2012 [33], and the DJIA, NASDAQ, NYSE, and S&P500 indices from

The asymmetric multifractal detrending moving average analysis (A-MF-DMA) may be easily expanded, as it employs moving averages as the trend function and determines the direction (or "sign") in the same way as the A-MF-DFA technique [55]. The asymmetric multiscale detrended fluctuation analysis was used to look at the hourly returns of California energy spot prices in 1999 and 2000 [56] by calculating the local exponents at different time scales.

Cao and colleagues created the asymmetric multifractal detrended cross-correlation analysis (MF-ADCCA) method, using the MF-X-DFA technique as an asymmetry extension (see Section 3.4). The method was used to find asymmetric cross-correlations between the daily returns of the SSEC index and six foreign exchange rates from 22 July 2005 to 13 January 2012 [357], the 5-min returns of the CSI 300 index spot and futures returns from 30 June 2011 to 7 June 2013 [358], the daily returns of gold spot prices on the Shanghai Gold Exchange (Au 99.95) and the New York Gold Exchange from 2 January 2003 to 27 April 2012 [59], and the EU ETS for carbon emissions from (Brent), From 4 January 2000 to 31 December 2014, the daily returns of the WTI price and eight foreign exchange rates (CAD, MXN, NOK, GBP, JPY, AUD, EUR, and KRW) against the USD [61], the Shanghai SSEC and four other indices (US S&amp;P 500, Germany DAX, India BSESN, and Brazil BVSP) from 1 January 2002 to 26 September 2014 [62], and the WTI crude oil pr from 1 January 2002 Chen and Zheng introduced the asymmetric joint multifractal analysis based on partition functions [364] as an extension of the MF-X-PF technique. (For further information, see Section 3.1.) This expansion is viable for financial volatility since the corresponding returns may be utilised to describe the signs in the segments.

**2.5.8. Multifractal diffusion entropy analysis**

The Shannon entropy [365–368] is used in the diffusion entropy analysis (DEA) for time series X(i)N.

i=1

This may be established using asset values as an example. For a given diffusion period or time scale s, we calculate the return series, which depicts the complete diffusion distance of s leap steps. The values of X(i, s)N for each s are covered by the Ns variance of the fluctuations X(i,1), which is independent of the time scale s. Using the formula, we count the number of Nj(s) X(i, s) returns that fall in the jth interval, as well as the probability (or frequency).

Nj(s) pj(s) ≡ N − s.

The Shannon entropy of the diffusion process is determined as

∆X(i,s)=X(i)−X(i−s), i=1,2,···,N−s,

i=1

length intervals that do not overlap (s). The interval length (s) is chosen as a fraction of the square root of the square root of the square root of the square root of the square root of the square root of the square root of the square root of the square

The scaling behaviour can be described as follows:

I(s) = −

pj(s)ln pj(s).

􏰗Ns

The scaling behavior is characterized by

I(s) ∼ δ ln s,

where is the scaling factor.

The DEA scaling exponent is equal to the Hurst index for fractional Brownian movements, that is, DEA = H, which does not hold for other processes such as Levy processes [67, 68]. It appears that in the case of financial assets, one should focus on returns to come near to = H; otherwise, the value would be bigger than 0.9 [69–71]. For small scales, the entropy I(s) deviates from the "theoretical" value, while for large scales, it converges to the theoretical value [68]. This problem can be avoided by employing the notion of integrated moments. [72]

The multifractal diffusion entropy analysis (MF-DEA), whose main result is a spectrum of scaling exponents (q) [73], is a simple expansion of DEA for multifractal analysis, in which the Shannon entropy is replaced by the Renyi entropy indicated in Eq (16). The multiscale multifractal diffusion entropy analysis may be constructed in the same way as the multiscale multifractal analysis using the MF-DFA method [74]. There are several faults in the MF-DEA technique [75]. We can see that (q) does not have a straightforward relationship with the well-known generalised dimensions Dq. As a result, the MF-DEA technique exposes multifractality in particular mappings of the original time series but not in the original time series itself. In the DEA or MF-DEA technique, the computation of the probabilities pj(s) has a considerable impact on the estimate of the Re nyi entropy and, as a result, the (q) spectrum [76]. The best option(s) must be picked in this way.

**3.5.9 Symbolization presentation and Renyi dimensions**

Xu and Beck calculated the Renyi dimensions, or generalised dimensions Dq [77], using financial returns using symbolic dynamics techniques. A number of symbolization techniques may be used to turn a financial time series X(i)N into a symbol sequence. Li and Wang proposed a strategy based on price change rates. i=1

which is quantified by an angle θi = arctan[(X(i + l) − X(i))/l],

This is between the numbers [90, 90]. We divide [90, 90] into 2n subintervals to represent the raw time series, which are normally fairly symmetric with regard to = 0. Li and Wang used four subintervals (90, 45), [45, 0], [0, 45], and [45, 90), as well as four symbols D, d, r, and R, to symbolise rapid falls, slow falls, slow increases, and fast rises of prices, respectively.

A similar symbolization method is used by Xu and Beck [377]. They proposed splitting the Xi (1, 1) return domain into 2n subintervals, each of which would be mapped into 2n symbols. When n = 1, the two subintervals are (1, 0) and [0,], respectively, and the corresponding symbols are d and u, which represent falling and growing prices. The four subintervals for n = 2 are (1, c1), [c1, 0], [0, c2), and [c2,], where c1 (0, 1) and c2 [0,] are parameters. It's worth noting that Xu and Beck used open intervals, which might lead to the removal of certain data points. In addition, no return can be less than one. For stock markets with price restrictions, the subintervals should be adjusted.

If an element of a given subinterval is I or Xi, the corresponding symbol determines the ith symbol S I, independent of symbolization. As a result, the raw time series is transformed to the symbol sequence Si. The symbol sequence is divided into Nk sections of equal length. For each k, we obtain (k) = (2n)k permissible symbol subsequences or configurations. The formula pj = p(C = Cj) may be used to compute the probability of each configuration C j of length k. Xu and Beck remembered the well-known Re nyi information [51].

## CHAPTER 4

#### PROPOSED TECHNOLOGY

###### 4.1 Emperical evidence of multifractal markets:

Over the last three decades, hundreds of empirical studies on the incidence of multifractality in financial markets have been done. In the 1990s, only a few notable research were published, and the vast majority of them employed the structural function methodology [18, 103, 113, 173, 690, 691] or the box-counting method [692]. Because of the development of fresh multifractal analytic approaches, empirical research surged in the twenty-first century.

A thorough and in-depth analysis of the literature reveals three trends. To begin with, the structure function approach was widely employed in early studies, while later research favoured multifractal detrended fluctuation analysis. Second, as one might anticipate, the return time series of stock market indexes garner the most attention, but many specialists also study foreign exchange rates and commodities. Finally, the bulk of the financial time series examined had a daily sample frequency. The second and third trends are caused by the availability of varied data sets. The experimentally obtained multifractal characteristics also provide no persuasive evidence for universality.

There are also only a few studies on economic variables other than financial variables, such as the monthly aggregate price indices of the US consumer price index (CPI) and producer price index (PPI) from 1975 to 2011 [693], the daily spot rates of VLGCs on the benchmark Persian Gulf (Ras Tanura) to Japan (Chiba) route from 3 January 1992 to 24 June 2009 [694], and the daily spot rates of ships in tanker markets from 27 January 1998 (or 1 July 2004 Multifractal analysis is typically used to financial time series since economic time series are frequently recorded monthly or quarterly, which is insufficient for multifractal analysis. As a result, the MF-SF technique is only used in one study [697] to determine NASDAQ monthly prices from 1984 to 2000.

In future research based on these findings, we feel it is less crucial to try to confirm the presence of multifractality in financial index returns. The evidence is irrefutable. Other financial considerations, such as liquidity measurements and multifractality comparisons across different assets, should be given more consideration. The financial processes and repercussions of multifractality, in particular, should be researched.

While the section is titled "asset returns," this is a slight misnomer because the multifractal aspects that are investigated are largely those of the absolute values of the returns, rather than signed returns in general. Because the absolute value of a return is a proxy for volatility, the multifractal aspects that have been studied are predominantly those of financial volatility across the asset classes mentioned below. The section below on volatility is about research that employed volatility estimated using various methods as direct inputs. There is no multifractality when the indicators are added. The return signals are usually eliminated, either expressly or implicitly, in most multifractal research.

**4.2 Volatilities:**

**4.2.1 Stock market:**

Researchers usually distinguish between two important areas when estimating the future of the insurance industry: macro forecasting and forecasting for individual insurance businesses/groups of organisations (Nazarova, 2018).

The method for performing insurance business development forecasts (individually and in groups) is exceptionally thorough and meets all of the study objectives. Only a few examples include discriminant analysis, logistic regression (Lytvin, 2013), linear, exponential modelling, and the application of fuzzy logic techniques (Serediuk, 2014), Methods such as probabilistic analysis can be used to forecast the financial status of an insurance firm or companies (Pozdniakova & Mamonova, 2011; Fouladvand & Darooneh, 2004), The methods employed include parametric econometric modelling (Proskurovych & Melnychuk, 2014), rating modelling (Gestel et al., 2007), and others. For example, in order to forecast the financial situation of insurance outcomes, the researchers themselves point out the difficulties of using these and other models, which make their application too specific, prohibiting outstanding forecasting quality under current conditions. Forecasting quality is influenced by the state of the national economy, random fluctuations in the insurance market, the characteristics of the insurance business, and other variables.

**4.2.2 Forex Exchange:**

In the building of a macroforecast of the insurance market, the insurance business, its themes, and goals are typically discussed (Lenten & Rulli, 2006). Comprehensive insurance sector projections are unusual. Their development is based on parametric ra- tios such as GDP dynamics/dynamics of collected premi- ums, payments dynamics/dynamics of completed insurance contracts, correlation and regression analysis, and so on (MAPFRE Economics Review, n.d.; OECD, 2020; Marsh, n.d.; Kozmenko et al., 2009; Shkolnyk et al., 2017). There have also been attempts to categorise countries based on the characteristics of their insurance markets. However, in addition to the market's volatility, the difficulty in finding quantitative patterns in its dynamics has a significant influence on the financial policy of regulators and governments. As a result, the researched tactics for predicting insurance business growth, which are frequently based on the deterministic paradigm of scientific thinking, provide inadequately predictable or short-term results. During a crisis, the accuracy of forecasting the insurance industry's dynamics as a whole decreases. Baluch et alassumptions .'s regarding the insurance market's evolution after the 2008 crisis, for example, and connected to the concept of systemic risk in insurance, were not realised. The inadequacy of existing parametric models for estimating the state of the insurance industry is highlighted by Kozmenko et al. 2009. (p. 30). It's no surprise that evaluations of the insurance industry's success only provide estimates for a fifth of the total. Simultaneously, the methodological tools of the indeterminist paradigm of scientific thinking, which enable qualitative findings in other domains of financial forecasting, are underutilised in calculating the prediction values of insurance market dynamics. Izzeldin (2007), for example, uses the variance of stochastic processes to simulate financial indicators. The author creates a multivariate exchange rate dynamics model that he uses to analyse daily random variations. According to Richards (2004), using fractal analysis in financial forecasting might help overcome issues like inhomogeneities at non-uniform intervals and scaling of proportion-al-symmetry linkages between fluctuations at different separation distances. This is characteristic of insurance market dynamics indicators, although not to the same degree as indicators for financial assets markets. To avoid disruptions in financial time series forecasting horizons, Schmitt et al. (2001) propose employing fractal analysis, calculating statistical temporal translational invariance for exchange rate time series based on multifractal fluctuations. Timashova and Skachko (2016). Stress the advantages of using R/S analysis to investigate fundamental aspects of time series, such as the presence and depth of long-term memory, trend resistance, and so on, when using financial time series (persistence). Like Kapecka (2013) and Sviridov and Nekrasova (2016), the authors of this work presume that stock price or financial instrument changes have a "long memory" and are self-similar. Dalton's dissertation (2006) compares the results of using fuzzy models to estimate stock price on time series with and without fractal analysis, emphasising that the fractal technique significantly improves forecasting quality.

**4.2.3 Commodities:**

Fractality in the market is created by a combination of global determinism and local unpredictability, which is present in the vast majority of financial markets. The "market fractality" hypothesis also offers the advantage of tolerating a certain number of flaws while maintaining system stability (Anderson & Noss, 2013). Not only do financial market prices have a "long memory" and are self-similar, but general market laws are also susceptible to erroneous perception. As a result, integrating the use of instruments in the approach for forecasting insurance market growth with the study of complexly organised natural genesis systems might improve the quality of the results. As a result, present methods for determining dynamical trends are fractional and do not give adequate estimates over long periods of time. At the same time, the insurance market in Ukraine is dynamic and unpredictable, making it necessary to identify the most constant trends. The purpose of this study is to identify trends in the insurance industry in Ukraine, as well as potential crisis sites.

**4.3 Data and Methods:**

Insurance companies' assets, UAH million (A); paid-up authorised capital, UAH million (K); formed insurance reserves, UAH million (R); gross insurance premiums, UAH million (GI Pr); gross insurance payments, UAH million (GIP); net insurance premiums, UAH million (GIP); net insurance premiums, UAH million (GIP); net insurance premiums, UAH million (GIP); net insurance premiums, UAH million (GIP); net insurance premiums, UAH million (NIP). The data for this study came from quarterly insurance statistics for the years 2014–2020 (Forinsurer, n.d.).

Indicators computed separately for each quarter using public information (number of concluded insurance contracts, gross insurance premiums, gross insurance payments, net insurance premiums, net insurance payments) were expressed cumulatively throughout the year. The GDP deflator is used to calculate the face value of indicators whose face value is affected by inflation (insurance companies' assets, paid-up authorised capital, formed insurance reserves, gross insurance premiums, gross insurance payments, net insurance premiums, UAH million, and net insurance payments, UAH million) (State Statistics Service of Ukraine, 2021).

To investigate trends in the dynamics of insurance market indicators, the parameters of approximation by linear, cyclic, exponential, exponential, and polynomial dependences were calculated and their reliability was tested using f-statistics. The cyclic component was determined via Fourier analysis. Since the average degree of dependability (more than 0,6 probability of approximation) was attained for the linear and cyclic components, a sophisticated series of dynamics was developed on the basis of two time variables (t and t/). This allowed for simultaneous consideration of the main type of dependence (linear dynamics of the corresponding variable's performance indicator t) and its cyclical fluctuations (cyclic dynamics of the corresponding variable's performance indicator measured in radians), as well as a high level of reliabil- ity (P = 0, 9999 by f – statistics) that outperformed any other approximation.

###### Diagram Description automatically generated

###### Fig.4.1 Algorithm

###### Multidimensional time series are used in worldwide practise to anticipate the evolution of the insurance business, it should be noted. Lenten and Rulli (2006, pp. 49-50) see time series of insurance indicators as being made up of different components (trend, cyclical component, seasonal, and irregular components), each of which is modelled separately in their paper on forecasting the development of life insurance in the Australian market. On the other hand, the outcomes of modelling for many components are analysed together.

###### A randomised R/S analysis (Gachkov, 2009) was performed to estimate the self-similarity of the dynamics, which may be used to measure the fractal size, persistence of the dynamics, and average cycle duration. This stage of study aims to demonstrate that dynamics persist and that stable patterns may be identified within them. Autocorrelation and the subsequent growth of the autocorrelation function are used to determine lags for deterministic processes and fractal length for inconsistent time series. Individual fractals were examined further for the dynamics of insurance market indicators using the characteristics of linear dependencies.

###### 4.3 Accuracy:

The nomenclature employed in accuracy evaluations of RS CNN classifications was influenced by the binary confusion matrix, with the class of interest referred to as the positive case and the background as the negative case. The number of true positive (TP) and true negative (TN) samples, which represent the number of negatives incorrectly mapped as positives and vice versa, and the two error categories of false positive (FP) and false negative (FN) samples, which represent the number of positives incorrectly mapped as negatives, are the four entries in the binary confusion matrix (Table 4). In statistical hypothesis testing, Type I errors are referred to as Type I errors, and Type II errors are referred to as Type II errors. Because of the many CNN classification classes, the digits TP, FP, TN, and FN might represent pixels, objects, or scenes. The majority of the time, TN is not specified for objects. As a result, the remaining three components for object detection and instance segmentation may be the only ones in a full confusion matrix.

Globalisation and the rise of global economic links have defined the close interaction of commodities and financial markets. As a consequence, the dynamics of different countries' financial markets are coordinated, and their development is mutually subordinated. Different countries' national markets are affected by the same (or similar) factors. Crises in some markets frequently lead to crises in others. All of this is true in the insurance industry. The evolution of national insurance markets has been affected by the features of insurance services as a core product, as well as the peculiarities of the current economic climate. Features of insurance service fulfilment dictate the much higher risks faced by insurance firms, as well as the need for more exact explanations of product costs, insurance portfolio structure and volume, and procedures and levels of inter-company interaction. In the current economic climate, economic agents are exposed to more risks, which leads to an increase in demand for insurance services. The fact that Ukraine's national market has not gone beyond the formation stage and is characterised by substantial concentration and structuring processes exacerbates these patterns of insurance market development. As a result, the country's increasing tendencies, which are distinguished by extreme volatility, are governed by the broad patterns of both the global and national insurance markets in Ukraine. The difficulty of obtaining estimates of probable crisis situations in the insurance sector's development is presently particularly critical.

Obtaining forecasts about the future state (direction) of certain financial processes may be done in a variety of ways (financial phenomenon). Different methodologies are adopted based on the activities that researchers must complete in order to make these predictions. When anticipating the route of processes in insurance, problems in defining the status of particular insurance businesses arise (groups of companies). Forecasting results are linked to one or two elements of how firms operate. As a result, a wide range of models are used, all of which yield high-quality outcomes, such as the prediction of possible disasters. Even with such limited output data and forecasting outcomes, it is challenging to develop models for long-term horizons since insurance revenues are frequently predictable but costs are unpredictable.

Quantitative patterns in the insurance market's overall evolution are far more difficult to discern. For example, fortuity has a much greater impact on overall market dynamics than it does on the standing of individual insurance businesses. The insurance industry's procedures, on the other hand, are still in the early stages of development. The insurance sector is constantly changing, with enormous ramifications across the financial system. Forecasts for the future state of the insurance sector are usually made over short time spans and contain major errors. This paper's trend models, which are based on linear and cyclic connections, are a good example. Due to their high degree of reliability, these models were unable to foresee probable moments of crisis in the Ukrainian insurance industry.

On the other hand, using fractal analysis and R/S analysis as part of it produced better results. Fractal analysis was chosen to identify probable crisis moments in the insurance industry's growth because of the high quality of other financial predictions made with its help. It was taken into consideration that existing applications of fractal analysis to the realisation of financial predictions are limited and largely relate to price fluctuations on the securities markets. As a result, determining the accuracy of fractal analysis in predicting difficulties in Ukraine's insurance market was crucial. Given the fractal nature of insurance market processes, further research into applying fractal analysis to predict potential crisis times is urgently needed. It is possible to get innovative findings for fractals of various orders on various indicators based on the fractal dimension. It's also feasible to figure out why insurance market dynamics have incomplete fractal dimensions. It seems sense to look for parametric linkages between indicators of insurance market growth in fractals of various dimensions in order to make market forecasts without identifying probable crisis moments.

As a result, extending the use of fractal analysis in the insurance industry (and maybe other financial markets) makes sense in order to get more comprehensive insights about future crisis times.

**CHAPTER-5**

**RESULTS AND DISCUSSION**

There are two points to make here: first, no one is manually counting autos. Because remote sensing equipment has a high enough resolution and computer image identification is advanced enough, software can distinguish between cars and asphalt to calculate the total number of cars at each store throughout the day, as well as the percentage of the parking lot that is full [17]. Second, the scale at which it is being implemented, for example, looks at all Wal-Mart locations in a certain state or region of the country; this data isn't nearly as relevant to tiny businesses or mom-and-pop shops. However, the number of individuals that visit a business isn't the only aspect that investors evaluate when assessing the feasibility of a firm [18]. The size of the overall market for cellular phones, for example, and the percentage of that market owned by Verizon, AT&T, and Sprint [19] are among the most powerful indications of future organisational performance. Another important metric for predicting a company's sustainability is its customer retention rate. The two steps of selling something are getting impressions on an object and turning those impressions into sales [20]. The second of the two procedures is the client retention rate. It's one thing to get customers to come into your store and look around, but how many of those customers really purchase what they see? This can be a good indicator of both the quality of your products and whether or not you're fulfilling demand [21]. Parking lot remote sensing is used to assess the potential for growth into other geographic locations as well as to determine how an existing firm is doing [22]. Satellite data on parking lots and traffic may be used to identify areas where new businesses may be in demand, as well as which locations will be profitable investments in the future [23]. Remote sensing of parking lots can be utilised to uncover investment opportunities in specific industries. On a local or regional scale, these satellite images may be utilised to assess the potential for economic or population development [24]. Parking lot remote sensing, on the other hand, can represent the state of an economy on a larger scale. Remote sensing isn't just for parking lots when it comes to economic analyses. According to recent articles on NPR and CNBC, investment firms like Lanworth use advanced crop remote sensing to distinguish between corn, soybeans, wheat, and other agricultural commodities in order to spot potential changes like diseases, floods, and fires that could impact future supply or demand [25]. Every day, new and novel ways to exploit satellite photos are devised by investing firms. It's worth emphasising that major investment firms are advanced and resourceful enough to use satellite data to make acquiring and selling decisions, whether from parking lots or elsewhere [26].

The financial time series was studied using fractal analysis. R/S analysis was used to identify persistent and antipersistent series. Long-term memory rows are more appealing to investors. The higher the Hurst index values, the more likely quotes will be predictable over time. When this is paired with the presence of discernible financial cycles, it is possible to miscalculate the best time to invest in the product in question, as well as the approximate timeframe during which these funds should be withdrawn to maximise profit. In this case, investing might provide returns over a long period of time.

For persistent series, non-periodic cycles have been defined. The prices for Brent oil cycles have intervals of 36, 448, and 2016 days, respectively, according to the statistics, and the financial range of the dollar versus the ruble has periods of 24 and 2016. Non-periodic cycles that last longer than 2016 days are possible. However, no inferences can be drawn because the available data on the time series spans a ten-year period. An R/S examination of the financial series detailing the stock quotes of the IT company ViaSat indicated that the series under consideration is random. The volatility of this series is also inconsistent. Low Hurst index values (less than 0.5) indicate frequent rate fluctuations, hinting that such a measure might be used for speculative tactics. Quotes will rise and decrease throughout time, allowing us to receive cash in a short amount of time (if we keep track of the rates). Traders will be interested in these series. They can profit in the short term by taking advantage of the financial series' short-term memory and volatility. Fractal analysis is one of the most effective methods for analysing financial data. For an investor, the persistent financial series, which may be considered for long-term investments, is the most fascinating. At this point in the study, numerous fractal analytic approaches were used to test the persistence hypothesis of the investigated series. There are also bicycles available. In the third instance, the Hirst index of 0.501 illustrates the unpredictability of the process under consideration. To put it another way, this signal shows that future values are independent of previous ones. This means that an investor who decides to invest in the shares of this firm will have a difficult time anticipating future quote behaviour. As a result, trading with such a commodity is extremely risky, making it inappropriate for both long-term and speculative tactics.

The insurance market is an important part of the financial system, and its correct operation protects people and businesses from the negative and stressful effects of today's unpredictable economic climate. The purpose of this study is to identify trends in the Ukrainian insurance industry as well as potential catastrophes in the future. According to the research, the insurance industry in Ukraine is constantly expanding, but it is also volatile and concentrated. The majority of indicators are cyclical, with cycles ranging from 4,66 to 14 quarters.

The fractal likeness and stability of the dynamics of Ukraine's insurance business were demonstrated by the randomised R/S-analysis. Fractal similarity was seen in six out of ten insurance industry indicators. Furthermore, it has been demonstrated that a trend break occurs while transiting from one fractal to the next. As a result, the emergence of crises on the Ukrainian insurance market is connected to the self-similarity of dynamics and the coincidence of bifurcation points for certain indicators of the country's growth. The fractal analysis revealed that the bifurcation of the number of finalised insurance contracts coincided with a partial crisis on the Ukrainian insurance market at the start of 2019.

According to calculations, the Ukrainian insurance sector's likely crisis periods are Q1-2 2017, Q1 2019, and Q1 2020, with only one of them occuring (Q1 2019). The first quarters of 2023 and 2026 may be the most likely dates for a crisis in Ukraine's insurance sector. The insurance market is one of the most important aspects of the financial system, and it is largely responsible for its stability and growth. Simultaneously, actions on the insurance market have an impact on real-world processes, as the insurance market compensates for risks and hazards that arise in the real world. The insurance industry is a unique study issue due to its variety across various forms of organisation. Within the national economy, the number of insurers on the market is limited; each insurer has its own priority insurance services list, as well as its own strategy for developing an insurance portfolio, insurance criteria, and so on. There is a territorially diverse insurance market due to the uneven concentration of the company's operations across the country. The socio-cultural idiosyncrasies of each area, each nation, which have a direct influence on policyholders' choice of insurance services, also promote heterogeneity in the global insurance industry. The relevance of insurance in the growth and capitalization of financial resource funds should also be examined. During the past 40 years, the emergence of insurance market crises has been connected to declining GDP (MAPFRE Economics Review, n.d.), and GDP dependency may be found in both established and emerging nations. At the same time, the features of the insurance market's growth may have an influence on the broader economy of the country. A issue in the insurance business might set off a financial disaster. For example, in 2008, the United States spent 182 billion dollars to preserve the well-known insurance company American International Group (OECD, 2020), whose debt holders included international investment funds.

Significant financial losses may have been prevented if the preconditions for the crisis had been detected. The insurance industry has been reorganised rather than collapsing as a result of the current economic crisis. Certain insurance products have grown in popularity, but their risks have increased dramatically. As a result, prices for certain insurance services increased: health insurance (+35 percent), property insurance (+20 percent), and cyber insurance (+35 percent in the US and 29 percent in the UK) (Marsh, n.d.), but average market growth declined for the first time in the first quarter of 2021 (Marsh, n.d.). At the same time, the global economic crisis has created a number of "waves" that can help the insurance industry (Amadeo, 2019): insurance company policyholders; direct and widespread education of policyholders on services that can be provided by insurance firms; sufficient and even excessive in- formation regarding hazards; new patterns of behaviour and risk categories are evolving; both insurers and consulting firms can use a wide range of analytical tools As a result of these and other changes, insurers may need to change their primary business model. However, they cause market asymmetry and instability, making it more difficult to predict the market's state. Global trends in the insurance market have been steadily increasing over the previous 10 years. The onset of the COVID-19 problem did not result in a market meltdown in most nations. Insurers' earnings and capital positions were not as strong in 2020 as they had been in previous years, but the market environment remained stable in general (Ogilvi, 2021). In 2019–2020, gross insurance premiums grew in both life and non-life insurance. Gross payments increased as well, with growth rates ranging from 80.2 percent in Russia to 11.7 percent in Malaysia, although insurance activity did not decline internationally. It should be noted that since 2001, favourable trends in dynamics have dominated the insurance industry. (pg. 51-52) (Kozmenko et al., 2009). The quarterly results of the theoretical approximation of the lines of dynamics of Ukraine's insurance business are presented below for the period 2014–2020. (Table 1). Compliance with the following dependences was verified for detecting trends in indicator dynamics: linear, logarithmic, power, exponential, and polynomial (with orders of magnitude ranging from 2 to 4), with a degree of dependability ranging from 0.37 to 0.95. For each of the examined indicators, a linear approximation with a cyclic component gives the highest degree of reliability.

In general, insurance market concentration indicators (number of registered insurers, number of life insurers, number of insurance contracts) are on the decline, supporting the preliminary conclusion of market concentration. Market size indicators, on the other hand, are rising. Simultaneously, the shapes of the theoretical approximation lines suggest that the market is potentially unstable. When a result, rates differ as insurance businesses' assets, paid-up share capital, and insurance reserves all grow at the same time. Excessive capital multiplication might cause the elasticity of expansion of paid-in authorised capital and insurance reserves to be much lower than that of insurance enterprises as a whole. Capital/assets, reserves/assets, and cyclical fluctuations are all in conflict with one other. The fluctuation durations of these indicators differ substantially from one another. Such dynamics also cause market instability due to the insecurity of insurance businesses. There is also considerable inconsistency in the patterns of gross/net insurance premiums and gross/net insurance payments. This disparity originates from the fact that these indicators' cyclical natures are vastly different, with net premium and payment swings being significantly larger than gross premium and payment swings.

As a result, the overall result of this study stage is as follows: The insurance sector in Ukraine is quickly growing and consolidating, followed by market volatility and insecurity among insurance companies and their financial results.

At this stage, the study's main result was that all indicators of Ukraine's insurance industry followed the same pattern. Simultaneously, the amount of time that each indicator's identical reliance is repeated changes substantially. The indicators of the number of registered insurers, life insurance businesses, and net insurance benefits had the shortest duration of change (4,66 quarters), whereas the indicators of insurance company assets and net insurance premiums had the longest (14 quarters). According to the findings, the onset of crises in Ukraine's insurance market is the consequence of cyclical components in the dynamics of numerous indicators matching their contingency patterns. As a result, numerical methods failed to establish a link between cyclical changes in dynamics and the onset of crisis periods in the Ukrainian insurance industry. "...all periodicities are "artefacts," not a characteristic of the process, but rather an aggregate conclusion that depends on the process itself, the length of the sample, and the economist's judgments," as B. Mandelbrot put it, and this is clearly applicable here (Mandelbrot, 1977).

Table

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Table.3.1 Result based on autocorrelated values

Based on this result and given the results of calculating autocorrelation in time series, the duration of first-order fractals for the indicators of the number of registered insurers, the number of life insurance companies, gross insurance payments, net insurance premiums, net insurance payments (12 quarters), and the number of insurance contracts was determined (7 quarters). For indicators of insurance business assets, produced insurance reserves, and gross insurance premiums, the length of the series prevented determining the duration of the first-order fractal. The same indications have a "long memory," lasting at least 14 quarters. The influence of time on the findings of creating quantitative patterns in the insurance market's dynamics has been noticed in other research. The dynamics of indicators in the growth of the German and Ukrainian insurance markets have yearly delays, according to Kozmenko et al. (2009, p. 53): 5 years for gross insurance premiums, per capita insurance premiums, and non-life insurance premiums; 4 years for the number of insurance businesses; 3 years for the ratio of gross insurance premiums to GDP; and 2 years for the amount of per capita insurance premiums.Chart, line chart

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Fig.5.1 Result comparision of empirical and theoretical values

When the dynamics of actual data are compared, practically all insurance market indices exhibit a substantial degree of similarity in first-order fractals. As an example, Figure 2 shows the dynamics of empirical values for gross insurance payments in first-order fractals.

Theoretical approximations of dynamics generated from actual data have a strictly linear structure for fractals of the first order, and vary the tilt angle to the axis x when transitioning from one fractal to another (Table 2). If market crises occur when a fractal transitions to another and is marked by large changes in dynamics, then significant changes in trends occur solely for the number of insurance contracts involved in the transition.

**CHAPTER-6**

**CONCLUSION**

Due to the development of new approaches and improved access to enormous volumes of financial data, the econophysics community has developed a particular interest in multifractal analysis for univariate and multivariate financial time series. Many new multifractal analytic methods are being created all the time, the bulk of which are variants on traditional methodologies. In this light, we suggest that massive numerical tests on mathematical models, as well as comparisons to other competing approaches, should be utilised to assess the performance of new methods. It's not easy to come up with new ideas that are more effective. Such techniques, on the other hand, are nonetheless beneficial in advancing our understanding. In multifractal analysis of empirical time series, selecting a technique is crucial. However, no one can agree on the most successful method. While comparing the outcomes of numerous ways for a single mathematical model is possible, the results for distinct mathematical models usually differ. In conclusion, given on current research, we may recommend techniques based on DFA, DMA, and wavelet leaders.

It's not easy to come up with new ideas that are more effective. Such techniques, on the other hand, are nonetheless beneficial in advancing our understanding. In multifractal analysis of empirical time series, selecting a technique is crucial. However, no one can agree on the most successful method. While comparing the outcomes of numerous ways for a single mathematical model is possible, the results for distinct mathematical models usually differ. In conclusion, given on current research, we may recommend techniques based on DFA, DMA, and wavelet leaders. The choice of the scaling range is particularly crucial and difficult for short time series, such as low-frequency financial time series. Despite the fact that numerous subjective (by eyeballing) and objective criteria have been provided, further study is required to determine the scaling range. This is due to the fact that existing criteria are particularly specific to the scenarios under consideration and hence lack universality.

It is both theoretically and practically vital to investigate the sources of apparent multifractality in empirical time series. The time series' linear and nonlinear correlations, as well as their non-Gaussian distribution, are often analysed macroscopically. Although it is unknown if nonlinearity is the primary source of intrinsic multifractality, a time series with only linear correlations and a fat-tailed distribution cannot produce multifractality. In empirical research, measures of multifractality strength are computed based on apparent multifractality. We feel that this is an issue that needs to be looked at more. Otherwise, an incorrect estimate of market efficiency, for example, may result. In order to construct multifractality time series, researchers have created a variety of agent-based models (ABMs) for financial markets. Agent-based modelling does, in fact, provide a valuable microscopic viewpoint for understanding the origins of multifractality in financial time series, which is regrettably underexplored. Research has revealed microscopic ABM principles that cause the development of multifractality in macroscopic time series. It also facilitates in the finding of a relationship between human behaviour and multifractality in financial markets. Most financial time series, such as returns, volatilities, trading volumes, recurrence intervals, and inter-trade durations, have a general consensus on multifractality's occurrence thus far. As a result, it is no longer a priority to confirm and add fresh information on the prevalence of multifractality in these financial quantities. Rather, one should pay attention to other financial indicators. Furthermore, common driving factors may be addressed in the study of multifractal cross correlations in order to extract the inherent multifractal cross-correlations between time series. Multifractal analysis' value in financial markets is demonstrated by other, more demanding applications of multifractality, such as asset pricing and risk management.

Another possibility is to use multifractal analysis to complex financial networks. On the one hand, financial time series' multifractal nature allows us to construct equity networks. Efforts have already been made in this direction. However, there is still more work to be done. On the other hand, we may look at the multifractal nature of complex financial networks and its implications. We expect to see this study path increase in the next years.

Against the ever-increasing number of hazards and dangers, the insurance business gathers the most effective protective weapons. The current status of the Ukrainian insurance business, on the other hand, indicates the country's vulnerability and reliance on unfavourable outside forces. In this regard, the study's purpose was to identify trends in the evolution of the Ukrainian insurance business as well as potential crisis areas. According to this report, even during times of crisis, the major patterns of dynamics of Ukraine's insurance industry have shown expansion and concentration. Simultaneously, the insurance market's expansion is accompanied with instability and the creation of systemic preconditions for disasters to occur. The primary indicators of the Ukrainian insurance business similarly follow a cyclical pattern, with cycles ranging from 4,66 to 14 quarters in duration. On the other hand, no association was found between the frequency of crisis moments in the history of the insurance market and the periodicity of theoretical dynamics lines when cyclic fluctuations in dynamics lines were investigated.

The formalisation of parametric relationships, as well as the construction of simple or intricate trends, was discovered to only allow for short-term forecasting of the Ukrainian insurance market's position.

According to the results of a randomised R/S-analysis, the dynamics of all indicators of Ukraine's insurance market (save for the indicator of paid authorised capital) are persistent and fractal-like, and they are linear inside each fractal of the first order. As a result, transitions from one fractal to the next (within the same order) might be bifurcation points. For the complete set of examined indicators of insurance market development, such a possible moment of bifurcation in the dynamics of the number of concluded insurance contracts occurred just once: in the first quarter of 2019, when the dynamics of the number of finished insurance contracts broke. At the same time, the Ukrainian insurance industry was in the throes of a crisis. There are three probable bifurcation moments throughout the same time period: Q1-2 2017, Q1 2019, and Q1 2020. According to the findings, the first quarters of 2023 and 2026 will be possible bifurcation moments for the insurance business in Ukraine.

The financial time series were analysed using the fractal analysis method. To find persistent and anti-persistent series, R / S analysis was performed. The long-term memory series is of particular interest to investors. The Hurst Index score reflects the degree to which the estimate is repeatable across time. When this is paired with the existence of a well-defined financial cycle, it is easy to make mistakes in judging when it is a good moment to invest in a product and when it is necessary to withdraw assets to maximise profits[18]. The investment may have a long-term influence in this case. A low score of less than 0.5 on a scale of one to ten.

According to the Hurst Index, prices vary often. This indicates that speculative techniques can be carried out with such tools. Estimates are always increasing and decreasing. This allows you to gain money in a short amount of time (as long as you stay on track)[19]. These series pique the curiosity of traders. You may capitalise from the financial series' short-term memory and volatility to gain short-term earnings. Fractal analysis is one of the most effective approaches for analysing financial data. Investors are primarily interested in the permanent financial series, which allows them to contemplate long-term investments. Multiple fractal analytic methodologies were utilised to assess the series's sustainability hypothesis at this phase in the investigation. There's also the cycle there[20]. The Hurst index was 0.501 in the third case, indicating that the process under examination was random. To put it another way, this indicator indicates that future values are unaffected by previous values. This means that future price behaviour for investors who choose to invest in the company's securities will be impossible to anticipate. As a result, dealing with such assets is extremely risky, making them unsuitable for long-term investments as well as speculative methods.

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